M.Sc. MATHEMATICS
First Semester
TOPOLOGY
(MSM - 104)

Duration: 3Hrs. **Full Marks: 70**

Part-A (Objective) =20
Part-B (Descriptive) =50

(PART-B: Descriptive)

Duration: 2 hrs. 40 mins. **Marks: 50**

Answer any *four* from *Question no.* 2 to 8
***Question no.* 1 is compulsory.**

1. (a) Let U consists of ϕ and all those subsets G of □ having property that to each $x \in G$, there exists $\varepsilon > 0$ such that $(x-\varepsilon, x+\varepsilon) \subset G$. Show that U is a topology for□ .

(b) Prove that **(6+4=10)**

(i) $[0,1] \sim (0,1)$, (ii) $[0,1] \sim [0,1)$, (iii) $[0,1] \sim (0,1]$

2. (a) Let $\Im$ be the collection of subsets of □ consisting of empty set ϕ and all subsets of the form $G_m = \{m, m+1, m+2, \ldots\}, m \in$ □ .

Show that $\Im$ is a topology for□ . What are the open sets containing 5?

(b) Consider the topology $\Im = \{X, \phi, \{a\}, \{a,b\}, \{a,c,d\}, \{a,b,c,d\}, \{a,b,e\}\}$ on the set $X = \{a,b,c,d,e\}$. List the members of the relative topology $\Im_Y$ on $Y = \{a,c,e\}$.

(5+5=10)

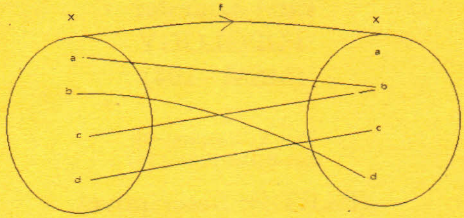
3. (a) Consider the topology $\Im = \{X, \phi, \{a\}, \{a,b\}, \{a,c\}\}$ on the set $X = \{a,b,c\}$. Find all limit points of the sets (i) $A = \{b,c\}$, (ii) $B = \{a,c\}$.

(b) What is a door space? Give one example.

(c) Consider the topology $\Im = \{X, \phi, \{a\}, \{a,b\}, \{a,c,d\}, \{a,b,c,d\}, \{a,b,e\}\}$ on the set $X = \{a,b,c,d,e\}$. List the neighbourhood of the point e. **(6+2+2=10)**

4. (a) Define base for a topology. Let $X = \{a,b,c,d,e\}$ and let $\mathbf{B}_* = \{\{a,b\}, \{b,c\}, \{a,d,e\}\}$. Find the topology on X generated by $\mathbf{B}_*$.

(b) Consider the topology $\Im = \{X, \phi, \{a\}, \{b\}, \{a,b\}, \{b,c,d\}\}$ on the set $X = \{a,b,c,d\}$. Let the function $f : X \to X$ defined by the following diagram.



Show that f is not continuous at c and d. **(5+5=10)**

5. (a) Prove that the property of a space being separable is a topological property.

(b) Show that the space $(\square, U)$ is T_3-space. **(5+5=10)**

6. (a) Let Y be subspace of topological space X and let $A \subset Y$. Then prove that A is compact relative to X if and only if A is compact relative to Y.

(b) Prove that closed subsets of compact sets are compact. **(7+3=10)**

7. (a) Consider the topology $\Im = \{X, \phi, \{a\}, \{b,c\}\}$ on the set $X = \{a,b,c\}$. Show that $(X, \Im)$ is aregular space.

(b) Prove that compact spaces have Bolzano Weistress property. **(5+5=10)**

8. (a) Prove that continuous image of a connected space is connected.

(b) Prove that every component of a topological space is closed. **(7+3=10)**

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REV-00
MSM/17/22 2016/12

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Duration: 20 minutes **Marks – 20**

(PART A - Objective Type)

I. Choose the correct answer: **1×20=20**

1. A set is called countable if it is

   a. Finite or denumerable b. Infinite or denumerable

   c. Denumerable d. None

2. A denumerable set has cardinality

   a. N b. N_0 c. α d. β

3. If $A \sim A'$, $B \sim B'$, $A \cap B = \varnothing$ and $A' \cap B' = \varnothing$, then

   a. $\#(A \cup B) = \#(A \cap B)$

   b. $\#(A \cup B) = \#(A' \cap B')$

   c. $\#(A \cup B) = \#(A \times B)$

   d. $\#(A \cup B) = \#(A' \times B')$

4. If $a \in$ □, then $\{a\}$ is a ***closed/open*** set in usual topology for□ . (Pick the correct one)

5. The interval $(0,1]$ is a neighbourhood of 0 under the usual topology of□ . State ***Yes** or **NO***.

6. Let (X, D) be any discrete topological space. Then derived set of A is

   a. Singleton Set c. Empty Set

   b. Non-empty Set d. None

7. Let $\Im$ and $\Im^{/}$ be two topologies for X which have a common base **B**, then

a. $\Im = \Im^{/}$ b. $\Im \subset \Im^{/}$ c. $\Im \supset \Im^{/}$ d. $\Im \neq \Im^{/}$

8. Let the real function $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$, then f is

a. Open b. Not open c. Closed d. Not continuous

9. The real line $\mathbb{R}$ with the usual topology is a

a. First countable c. Separable

b. Non Separable d. Compact

10. An indiscrete space is not a T_0 - space. State ***Yes** or **NO***.

11. Every T_0-space is T_1-space. State ***Yes** or **NO***.

12. The property of a space being a normal space is

a. Hereditary property c. Topological property

b. Both hereditary and topological property d. None

13. Every T_2-space is T_1-space. State ***Yes** or **NO***.

14. Consider the topology $\Im = \{X, \phi, \{a\}, \{b\}, \{a,b\}\}$ on the set $X = \{a,b,c\}$. Then $(X, \Im)$ is a

a. Regular space c. Normal space

b. T_4-space d. T_1-space

15. Closed subsets of a compact set are

a. Compact b. Not compact c. Closed d. Open

16. Consider the following class of open intervals

$$A = \left\{(0,1), \left(0, \frac{1}{2}\right), \left(0, \frac{1}{3}\right), \left(0, \frac{1}{4}\right), \ldots\ldots\right\}.$$

Then A has ***FIP/empty*** intersection. (Pick the correct one)

17. Every closed and bounded interval in □ is

a. Compact b. Not compact c. Finite d. Infinite

18. Cantor's set Γ is

a. Not compact b. Compact c. Open d. None

19. A connected space has ………component.

a. 1 b. 2 c. 3 d. 4

20. Two disjoint sets are separated. State ***Yes** or **NO***.

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