MSE
First Semester
Engineering Mathematics & Statistics
(MSE-01)

Duration: 3Hrs. Full Marks: 70

(PART-B: Descriptive)

Duration: 2 hrs. 40 mins. **Marks: 50**

1. Answer the following questions (Any five) **$5 \times 2 = 10$**

(a) Find a unit normal vector to the surface $x^2 + 3y^2 + 2z^2 = 6$ at the point $P(2, 0, 1)$

(b) Find the divergence of the vector filed

$$\vec{V} = (x^2 - y^2)\hat{\imath} + 2xy\hat{\jmath} + (y^2 - xy)\hat{k}$$

(c) If $F\{F(x)\} = f(\lambda)$ then show that $F\{F(x)cosax\} = \frac{1}{2}\{f(\lambda + a) + f(\lambda - a)\}$

(d) State and prove the linearity property of Laplace transform

(e) Obtain the Laplace transform of $e^{-3}cosh2x$.

(f) Find the Z-transform of the unit step function

$$U(k) = \begin{cases} 1, & if\ k = 0 \\ 0, & if\ k \neq 0 \end{cases}$$

(g) Two cards are drawn at random from a deck of 52 cards. Find the probability that both are spade.

2. Answer the following questions (Any five) $5 \times 3 = 15$

(a) Define curl of a vector. Show that gradient field describes an irrotational motion.

(b) Prove that, $\nabla.\left(\nabla \times \vec{V}\right) = 0$, for every vector$\vec{V}$.

(c) Solve: $\int_0^{\propto} F(x)cos\lambda x\, dx = e^{-\lambda}$

(d) If F is a periodic function of period ω that is $F(t + n\omega) = F(t)$, where n is a positive integer. Show that $L\{F(t)\} = \frac{\int_0^{\omega} e^{-\lambda x}F(x)dx}{1-e^{-\lambda\omega}}$

(e) Obtain the Laplace transform of $e^{-2t}\sinh 5t$.

(f) Find the Z-transform of $cos\alpha k, k \geq 0$.

(g) Suppose A and B be two events with $P(A) = 0.6$, $P(B) = 0.3$ and $P(A \cap B) = 0.2$. Find the probability of the following cases:

(i) B doesn't occur.

(ii) A or B occur

(ii) Neither A nor B occur.

3. Answer the following questions (Any 5) $5 \times 5 = 25$

(a) Write down the statement of Strokes theorem. Using this or otherwise, evaluate

$$\int_C [(2x - y)dx - yz^2dy - y^2zdz]$$

Where c is the circle$x^2 + y^2 = 1$, corresponding to the surface of the sphere of unit radius.

(b) Obtain the Fourier series of the following function

$$F(x) = \begin{cases} -x & if -\pi < x < 0 \\ x & if \quad 0 \leq x < \pi \end{cases}$$

and hence find the sum of the Fourier series as $x = \pi/_2$

(c) Using convolution theorem, obtain the inverse Laplace transform of

$$\frac{7}{(\lambda-7)(\lambda^2+25)}$$

(d) Solve the difference equation by using Z-transform

$$6y_{k+2} - y_{k-1} - y_k = 0, k \geq 2, y_{(0)} = 0, y_1 = 1$$

(e) Define probability density function. If

$$f(x) = Ae^{-3x} - 1, \qquad 0 \leq x \leq 1$$

is a probability density function of a random variable X, the find the value of A. Also find E(X).

(f) State Parsevel's identity for Fourier cosine transform. Using it, prove that

$$\int_0^{\infty} \frac{dx}{(a^2+x^2)(b^2+x^2)} = \frac{\pi}{2ab(a+b)}$$

(g) Define irrotational vector and scalar potential. Show that

$$\vec{F} = (y^2 + 2xz^2)\hat{\imath} + (2xy - z)\hat{\jmath} + (2x^2z - y + 2z)\hat{k}$$

is irrotational and hence find its scalar potential.

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MSE
First Semester
ENGINEERING MATHEMATICS & ~~STATICS~~ STATISTICS
(MSE-01)

PART A: Objective

Duration: 20 minutes **Marks - 20**

(Choose the correct option and make a circle around the corresponding number)

1. The unit vector along the vector $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ is

   (a) 1 (b) $\frac{1}{\sqrt{3}}(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$

   (b) $\frac{1}{3}(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$ (d) $\frac{1}{3}$

2. If $\vec{r}$ is a irrational vector then

   (a) $\Delta \times \vec{r} = 0$ (b) $\Delta . \vec{r} = 0$

   (c) $\Delta\vec{r} = 0$ (d) $\Delta \times (\Delta\vec{r}) = 0$

3. Let $\phi(x, y, z) = c$ be a family of surfaces. Then which of the following is not true

   (a) $\Delta\phi$is a vector.

   (b) $\Delta\phi$ is a vector normal to the surface $\phi(x, y, z) - c$

   (c) $\Delta\phi . \hat{d}$ is the directional derivative of ϕ in the direction of $\vec{d}$.

   (d) $\Delta\phi$ is a vector tangent to the surface $\phi(x, y, z) = c$

4. If $\vec{a}$and $\vec{b}$ are two vectors such that $\vec{a} \perp \vec{b}$ then

   (a) $\vec{a}.\vec{b} = 0$ (b) $\vec{a} \times \vec{b} = 0$

   (c) $\vec{a}.\vec{b} = |a||b|$ (d) $\vec{a} \times \vec{b} = \propto$

5. Which of the following is an odd function

   (a) $f(x) = cos2x$

   (b) $g(x) = cos2x + 3x^2$

   (c) $h(x) = sin3x$

   (d) $k(x) = xsin3x$

6. If $F\{F(x)\} = f(\lambda)$, then $F\{F(ax)\}$ is given by

   (a) $\frac{1}{a} f(\frac{\lambda}{a})$

   (b) $f(\frac{\lambda}{a})$

   (c) $f(\lambda)$

   (d) $e^{i\lambda a} f(\lambda)$

PTO....

7. The convolution of two functions $F(x)$ and $G(x)$ defined on $(\propto, \propto)$ is

(a) $\int_{-\propto}^{\propto} F(x)G(x-u)du$

(b) $\int_{-\propto}^{\propto} F(u)G(x-u)du$

(c) $\int_{-\propto}^{\propto} F(x)G(x+u)du$

(d) $\int_{-\propto}^{\propto} F(u)G(x+u)du$

8. If $F\{F(x)\} = f(\lambda)$ and $F\{G(x)\} = g(\lambda)$, then Parsevals identity states that

(a) $\frac{1}{2\pi}\int_{-\propto}^{\propto} f(\lambda)g(\bar{\lambda})d\lambda = \int_{-\propto}^{\propto} F(\bar{x})g(x)dx$

(b) $\frac{1}{2\pi}\int_{-\propto}^{\propto} f(\lambda)g(\bar{\lambda})d\lambda = \int_{-\propto}^{\propto} F(\bar{x})g(\bar{x})dx$

(c) $\frac{1}{2\pi}\int_{-\propto}^{\propto} f(\bar{\lambda})g(\bar{\lambda})d\lambda = \int_{-\propto}^{\propto} F(x)g(x)dx$

(d) $\frac{1}{2\pi}\int_{-\propto}^{\propto} f(\lambda)g(\bar{\lambda})d\lambda = \int_{-\propto}^{\propto} F(x)g(\bar{x})dx$

9. The value of $\int_0^{\propto} \sin 2t\, \delta(t-\pi)dt$ is

(a) -1 (b) 1

(c) 0 (d) $\propto$

10. Laplace transform of $\cos \omega t$ is

(a) $\frac{\omega}{\lambda^2 + \omega^2}$

(b) $\frac{\lambda}{\lambda^2 + \omega^2}$

(c) $\frac{\omega}{\lambda^2 - \omega^2}$

(d) $\frac{\lambda}{\lambda^2 + \omega^2}$

11. Laplace inverse transform of $\frac{1}{\lambda^2}$ is

(a) 1 (b) t

(c) $\frac{t^2}{2}$ (d) t^2

12. If $L\{F(t)\} = f(\lambda)$, then $L\{e^{at}F(t)\}$ is

(a) $f(\lambda + a)$

(b) $f(\lambda^2 - a^2)$

(c) $f(\lambda - a)$

(d) $\frac{1}{a} f(\frac{\lambda}{a})$

PTO...

13. $L^{-1}\{\frac{1}{\lambda^2-7}\}$is

(a) $sin7t$

(b) $sin\sqrt{7}t$

(c) $\frac{1}{\sqrt{7}}sin\sqrt{7}t$

(d) $\frac{1}{\sqrt{7}}sinh\sqrt{7}t$

14. The corresponding to $k = -2$ and $k = 2$ of

(a) 7 and 0 (b)10 and 3

(c) 0 and 7 (d)3 and 10

15. Z-transform of the unit impulse $\delta(k) = \begin{cases} 1, & if\ k - 0 \\ 0, & if\ k \neq 0 \end{cases}$ is

(a) 0 (b) $\frac{z}{z-1}$

(c) 1 (d) -1

16. The poles of $\frac{z^2+3z-1}{(z^2-1)(z-2)}$ are

(a) $1,\ 2$ (b) $-1, 1, 2$

(c) $-i, i, 2$ (d) $0, 1, 2$

17. The total number of Possible outcomes of throwing three dice simultaneously

(a) ${}^6C_3$

(b) ${}^6P_3$

(c) $6 \times 6 \times 6$

(d) 3×6

18. Which of the following is not true for any two independent events A and B

(a) $P(A \cap B) = P(A)P(B)$

(b) $P(A/B) = A$

(c) $P(B/A) = B$

(d) $P(A \cap B) = P(A) + P(B)$

19. The mean and variance of the Poisson distribution $P(n, \lambda) = \frac{\lambda^n e^{-\lambda}}{n!}$ are respectively

(a) n, λ (b) λ, λ

(c) $\lambda, n\lambda$ (d) $n\lambda, \lambda^2$

20. If A and B are two mutually exclusive events, and $P(A) = 0.2,\ P(A \cup B) = 0.58$, then $P(B)\ is$

(a) 0.8 (b) 0.78

(c) 0.30 (d) 0.38

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