

**BACHELOR OF COMPUTER APPLICATION**  
**Second Semester**  
**Discrete mathematics**  
**(BCA - 09)**

**Duration: 3Hrs.**

**Full Marks: 70**

**(PART-B: Descriptive)**

**Duration: 2 hrs. 40 mins.**

**Marks: 50**

**1. Answer the following (Any five)**

**2x5=10**

- a. The sum of the degrees of all vertices of a graph is an even integer.
- b. Does there exist a simple graph with five vertices having degree 2, 2,4,4,4? Justify.
- c. Make the truth table of the following

$$(p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$$

- d. Show that  $f(x) = 7x + 5$  is one-one, where  $f$  is a function from  $R$  to  $R$ .
- e. If  $f: R \rightarrow R_+$  is a function, define by  $f(x) = e^x$ , then show that  $f$  is homomorphism. (Where  $R$  is the additive group of real numbers &  $R_+$  is the multiplicative group of real numbers)
- f. If  $A = \{2,3,4,5\}$ ,  $B = \{4,5,6\}$  then find  $A \setminus B$ .
- g. If  $R$  is a ring prove that

i.  $a(-b) = -ab$

ii.  $a(b-c) = ab-ac$

*PTO....*

2. Answer the following (Any five)

3x5=15

- a. If  $I$  be the set of integers and  $R = \{(x, y): x - y \text{ is divisible by } 5; x, y \in I\}$ . Then show that  $R$  is an equivalence relation on  $I$ .
- b. Prove that a non-empty subset  $H$  of a group  $G$  is a subgroup of  $G$  if  $ab^{-1} \in H$ ,  $\forall a, b \in H$
- c. Define the homomorphism of a group. If  $f: G \rightarrow G'$  is a homomorphism, Then prove That  $f(a^{-1}) = f(a)^{-1} \quad \forall a \in G$
- d. How many vertices are there in a graph with 15 edges if each vertex is of degree 3?
- e. Prove that every connected graph has at least one spanning tree.
- f. Determine whether the following statements are tautology, contradiction or Contingents.

$$p \rightarrow (q \rightarrow (p \wedge q))$$

- g. Draw a circuit diagram of the Boolean expression

$$xyz' + xy'z + x'y'z'$$

3. Answer the following (Any five)

5x5=25

- a. Prove that if  $f$  is a homomorphism from  $G$  into  $G'$  with kernel  $K$ , then  $K$  is normal.
- b. Construct a logical circuit of the following Boolean expressions
  - i.  $x'yz + xyz' + x'yz' + x'y'z + x'y'z'$
  - ii.  $xy' + y(x' + y)$
- c. Prove that a tree with  $n$  vertices contains exactly  $n - 1$  edges.
- d. Prove that if a connected graph  $G$  is Eulerian, then every vertex of  $G$  has even degree.

- e. Show that the set of all positive rational numbers forms an abelian group under the composition defined by  $a * b = (ab)/2$
- f. Prove that a ring  $R$  is without zero divisors if and only if the cancellation laws hold in  $R$ .

g. If  $R$  is a relation in  $N \times N$  defined by

$$(a, b) R (c, d) \text{ if } ad = bc$$

Prove that  $R$  is an equivalence relation in  $N \times N$  (where  $N$  is the set of natural numbers).

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*(The figures in the margin indicate full marks for the questions)*

Duration: 20 minutes

Marks – 20

**PART A- Objective Type**

**I. Answer each of the following:**

**1×20=20**

1. If  $A = \{2n : n \in \mathbb{N}, n < 6\}$ , then  $A \cup N$  is

- a. A                      b. R                      c. N                      d. None of these

2. Which of the following is false?

- a.  $A^c \cap B^c = A \setminus B$                       b.  $A \cap B^c = A \setminus B$   
c.  $A^c \cap A = U$                       d. None of these

3. Every cyclic group is abelian.

**True/False**

4. Intersection of two subgroups is a subgroup.

**True/False**

5. In a graph there are even number of vertices of odd degree.

**True/False**

6. A walk with no repeated edges is called a trail.

**True/False**

7. For a Boolean algebra which of the following is false

- a.  $a \cdot 1 = a$                       b.  $a + 1 = 1$   
c.  $a + a = a$                       d.  $a \cdot a = 1$

8.  $a + b' = 1$  if and only if

- a.  $a + b = a$                       b.  $a + b = b$   
c.  $a + b = 0$                       d.  $a + b = 1$

9. The simplest Boolean expression of  $\{(x + y)(x + y')y + x\}x + yy'$  is

- a. x                      b. x + y                      c. y                      d. x'

10. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = 2x + 3$ , then
- $f$  is one –one
  - $f$  is onto
  - $f$  is both one-one & onto
  - None of these
11. A subset  $H$  of a group  $G$  is a subgroup of  $G$  if
- $ab^{-1} \in H, \forall a, b \in H$
  - $ab \in H, \forall a, b \in H$
  - $ab^{-1} \in H, \forall a, b \in G$
  - $ab \in H, \forall a, b \in G$
12. If  $f$  is a homomorphism, then it will be an isomorphism if
- $f$  is one –one
  - $f$  is onto
  - $f$  is both one-one& onto
  - None of these
13. A ring  $R$  is without zero divisors if for  $a, b \in R$
- $ab=0 \Rightarrow a=0$  or  $b=0$
  - $ab=0 \Rightarrow a \neq 0$  or  $b \neq 0$
  - $ab=0 \Rightarrow a \neq 0$  or  $b=0$
  - None of these
14. If  $n \in \mathbb{N}$ , then  $1+2+3+\dots+n$  is
- $\frac{n(n-1)}{2}$
  - $\frac{n(n+1)}{2}$
  - $\frac{n}{2}$
  - $\frac{n^2}{2}$
15. If  $f(x) = 3x + 4$ , then  $f^{-1}$  is
- $\frac{x-4}{3}$
  - $x - 4$
  - $4x - 3$
  - None of these
16. The identity element of a group is
- unique
  - May not be unique
  - More than one
  - None of these
17. The set of natural number is a group w.r.t. addition. **True/False**
18. Every one-one function is onto. **True/False**
19.  $\pi$  is a rational number. **True/False**
20. A graph is simple if it contains a loop. **True/False**

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