Chapter VII

Roseland Approximation for Heat generation/absorption on free convective radiating fluid with Soret effect

7.1 Introduction:

Heat generation is important in the context of exothermic or endothermic chemical Reaction and because of its numerous applications it has been a subject of interest of many researchers like Beg *et al.* (2005) studied the hydromagnetic oscillating heat transfer in a Darcian regime with heat generation/absorption using a perturbation technique. Radiative and free convective effects on MHD flow through a porous medium with periodic wall temperature and heat generation and absorption investigated by Sharma *et al.* (2014). Deka and Bhattacharya (2011) explained unsteady free convective Couette flow of heat generating/absorbing fluid in porous medium. Rajput and Sahu (2011) described radiation effects on steady hydromagnetic flow of a viscous fluid through a vertical channel in a porous medium with heat generation or absorption. Jha and Mussa (2012) studied unsteady natural convection Couette flow of heat generating/absorbing fluid between vertical parallel plates filled with porous material. Vajravelu and Hadjinicolaou (1997) studied the convective heat transfer in an electrically conducting fluid near an isothermal stretching sheet and they studied the effect of internal heat generation or absorption. Shanker *et al.* (2010) studied about the numerical solution of unsteady two-dimensional, laminar boundary layer flow of a viscous, incompressible, electrically conducting fluid along a semi-infinite vertical plate in the presence of thermal and concentration buoyancy effects under the influence of uniform magnetic field applied normal to the flow. Kinyanjui *et al.* (2001) presented simultaneous heat and mass transfer in unsteady free convection flow with radiation absorption past an impulsively started infinite vertical porous plate subjected to a strong magnetic field. Mishra *et al.* (2015) presented a solution for the transient free convective flow of a viscous and incompressible fluid between two vertical walls as a result of heat and mass transfer. Hady *et al.* (2006) studied the problem of free convection flow along a vertical wavy surface embedded in electrically conducting fluid saturated porous media in the presence of internal heat generation or absorption effect.

When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of more intricate in nature. Mass fluxes influenced by temperature gradient are termed as Soret or thermal diffusion effect. The research work relevant to this field are Sengupta *et al.* (2015) analyzed a basic theoretical fluid model depicting the parametric effect of the Peclet numbers on a two dimensional chemically reactive heat and mass transfer flow past an oscillating plate with Soret and first order chemical reaction effects. Bhavana *et al.* (2013) presented an important work on free convective unsteady MHD flow in a vertical plate with heat source, thermo diffusion (Soret effect) and the influence of the thermal radiation on hydromagnetic for a viscous fluid past a semi-infinite vertical moving porous plate embedded in a porous medium. Moreover Sengupta and Ahmed (2014) investigated the Soret effect with chemical reaction in case of MHD free convective dissipative flow in velocity slip regime. Ahmed *et al.* (2013) investigated the Soret effect in MHD free convection flow. Alam and Rahman (2006) presented Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction embedded in a porous medium for a hydrogen-air mixture as the nonchemical reacting fluid pair. Osalusi *et al.* (2008) studied numerically the effect of thermal-diffusion and diffusion-thermo on combined heat and mass transfer of a steady hydromagnetic convective and slip flow due to a rotating disk with viscous dissipation and Ohmic heating. Beg *et al.* (2009) investigated the combined effects of Soret and Dufour diffusion and porous impedance on laminar magnetohydrodynamic mixed convection heat and mass transfer of an electrically-conducting, Newtonian, Boussinesq fluid from a vertical stretching surface in a Darcian porous medium under uniform transverse magnetic field.

It is worth mentioning that radiation effects on the convective flow are very useful in the context of space technology, in engineering processes, process involving high temperature, industrial areas and for the design of pertinent equipment like nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such areas such as heating and cooling chambers, fossil fuel combustion. Keeping in view these important applications many authors studied it such as Cess (1966) investigated about the interaction of thermal radiation with free convection heat transfer. Chamkha *et al.* (2001) examined radiation effects on a free convection flow past a semi infinite vertical plate with mass transfer. Kim (2000) discussed unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Khatun and Ahmed (2014) presented an analytical solution for the steady magnetohydrodynamic laminar mixed convection heat and mass transfer flow of viscous electrically conducting fluid past a vertical permeable surface embedded in a Darcian porous regime with thermal radiation and chemical reaction effects. Soundalgekar and Takhar (1993) investigated radiation effects on free convection flow of a gas past a semi-infinite flat plate. Sengupta and Sen (2013) investigated the thermal radiation on the free convective heat and mass transfer flow in presence of heat generation and thermo-diffusion effects. An investigation has been performed for unsteady Magnetohydrodynamic boundary layer flow and heat transfer through a Darcian porous medium bounded by a uniformly moving semi-infinite isothermal vertical plate in presence of thermal radiation by Khatun and Ahmed (2015). An analytical solution of MHD free convective, dissipative boundary layer flow past a vertical porous surface in the presence of thermal radiation, chemical reaction and constant suction, under the influence of uniform magnetic field was studied by Raju *et al.* (2014). The effects of hall current, chemical reaction and radiation on a free convection flow bounded by a vertical surface embedded in porous medium under the influence of uniform magnetic field was studied by Reddy *et al.* (2012). Sharma *et al.* (2014) studied the effect of magnetic field and radiating heat transfer on unsteady free convection viscous incompressible electric conducting fluid past a vertical surface in a rotating porous medium. Mohammed Ibrahim *et al.* (2012) presented the radiation and chemical reaction effects on MHD free convection flow past a moving vertical plate. Samad *et al.* (2013) investigated the effects of MHD free convection heat transfer of power-law non-Newtonian fluids along a stretching sheet.

The objective of the present problem is to study about the effects of thermal diffusion (Soret effect) and heat generation/absorption of an unsteady flow of viscous, incompressible, electrically conducting fluid past a semi-infinite vertical moving porous plate embedded in a uniform porous medium in the presence of thermal radiation.

7.2 Mathematical Formulation:

In the present problem we consider free convection two dimensional unsteady flow of laminar, incompressible, viscous, electrically conducting, heat generation/absorption fluid past a semi-infinite vertical moving porous plate embedded in a uniform porous medium subjected to transverse magnetic field in the presence of a pressure gradient taking into account the thermal diffusion (Soret effect) and thermal radiation effects. The coordinate system is chosen such that $\bar{x}-$ axis is taken along the porous plate in the upward direction and $\bar{y}-$axis normal to it. The fluid is assumed to be a gray, absorbing-emitting but non-scattering medium. The radiative heat flux in the $\bar{x}$ $-$direction is considered negligible in comparison with that in the $\bar{y}$ $-$direction Sparrow and Cess (1995). It is assumed that there is no applied voltage of which implies the absence of an electric filed. The transversely applied magnetic field and magnetic Reynolds number are very small and hence the induced magnetic field is negligible Cowling (1957). Viscous and Darcy resistance terms are taken into account the constant permeability porous medium. The MHD term is derived from an order-of magnitude analysis of the full Navier-stokes equation. It is assumed here that the hole size of the porous plate is significantly larger than a characteristic microscopic length scale of the porous medium. We regard the porous medium as an assemblage of small identical spherical particles fixed in space, following Yamamoto and Iwamura (1976). A homogeneous first-order chemical reaction between the fluid and the species concentration has been considered. The chemical reactions are taking place in the flow and all thermo physical properties are assumed to be constant of the linear momentum equation which is approximation. The fluid properties are assumed to be constants except that the influence of density variation with temperature and concentration has been considered in the body-force term. Due to the semi-infinite place surface assumption furthermore, the flow variable are functions of $\bar{y}$ and $\bar{t}$ only. The governing equation for this investigation is based on the balances of mass, linear momentum, energy, and concentration species. Under all the assumptions, the flow is depicted mathematically as

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \ , \tag{7.2.1}$$

$$\rho\left(\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = \frac{\partial \bar{p}}{\partial \bar{x}} + \mu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \rho g - \frac{\mu}{\bar{K}}\bar{u} - \sigma B_0^2 \bar{u} \ , \tag{7.2.2}$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{\kappa}{\rho C_p}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - \frac{1}{\rho C_p}\left(\frac{\partial \bar{q}_r}{\partial \bar{y}}\right) - \frac{Q_0}{\rho C_p}(\bar{T} - \bar{T}_\infty) \ , \tag{7.2.3}$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{C}}{\partial \bar{y}} = D\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + D_T\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \ , \tag{7.2.4}$$

where $\bar{x}$, $\bar{y}$ and $\bar{t}$ are the dimensional distances along and perpendicular to the plate and dimensional time respectively. $\bar{u}$ and $\bar{v}$ are the components of dimensional velocities along $\bar{x}$ and $\bar{y}$ directions, ρ is the fluid density, μ is the viscosity, C_p is the specific heat at constant pressure, σ is the fluid electrical conductivity, B_0 is the magnetic induction, $\bar{K}$ is the permeability of the porous medium, $\bar{T}$ is the dimensional temperature, D is the coefficient of chemical molecular diffusivity, D_T is the coefficient of thermal diffusivity, $\bar{C}$ is the dimensional concentration, κ is the thermal conductivity of the fluid, g is the acceleration due to gravity and , $\bar{q}_r$, R are the local radiative heat flux, the reaction rate constant respectively. The term

$Q_0(\bar{T} - \bar{T}_\infty)$ is assumed to be amount of heat generated or absorbed per unit volume, Q_0 is a constant, which may take on either positive or negative values. When the wall temperature $\bar{T}$ exceeds the free stream temperature $\bar{T}_\infty$, the source term when $Q_0 > 0$ and heat sink when $Q_0 < 0$. The magnetic and viscous dissipations are neglected in this study. It is assumed that the porous plate moves with a constant velocity $\bar{u}_p$ in the direction of fluid flow and the free stream velocity $\bar{U}_\infty$ follows the exponentially increasing small perturbation law. In addition, it is assumed that the temperature and concentration at the wall as well as the suction velocity are exponentially varying with time.

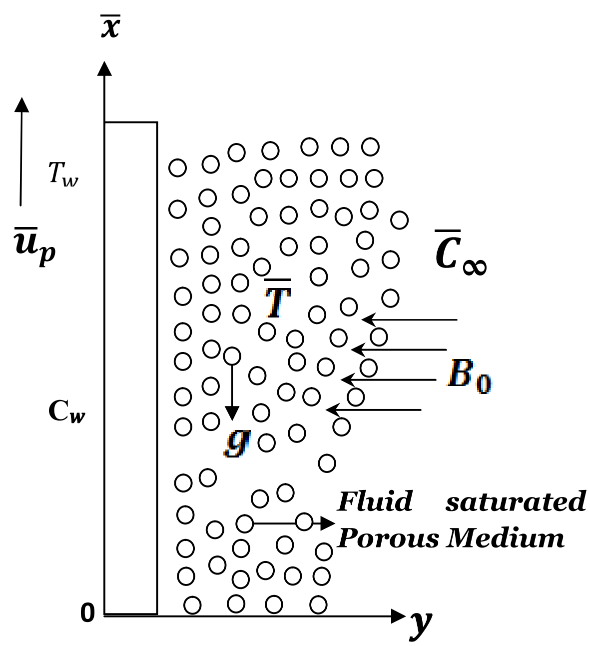

Figure 7.2(a): Physical model and coordinate system

The boundary conditions for the velocity, temperature and concentration fields are given as follows:

$$\bar{u} = \bar{u}_p \,, \bar{T} = \bar{T}_w + \varepsilon(\bar{T}_w - \bar{T}_\infty)e^{\bar{n}\bar{t}} \,, \bar{C} = \bar{C}_w + \varepsilon(\bar{C}_w - \bar{C}_\infty)e^{\bar{n}\bar{t}} \,, \quad at\ \bar{y} = 0 \quad (7.2.5)$$

$$\bar{u} \to \bar{U}_\infty = U_0\left(1 + \varepsilon e^{\bar{n}\bar{t}}\right), \bar{T} \to \bar{T}_\infty, \bar{C} \to \bar{C}_\infty \quad as\ \bar{y} \to \infty \,, \qquad (7.2.6)$$

where $\bar{T}_w$ and $\bar{C}_w$ are the wall dimensional temperature and concentration, respectively, $\bar{C}_\infty$ is the free stream dimensional concentration, U_0 and $\bar{n}$ are constants.

It is clear from equation (7.2.1) that the suction velocity at the plate surface is a function of time only and under this assumption it takes the following exponential form:

$$\bar{v} = -v_0\left(1 + \varepsilon A e^{\bar{n}\bar{t}}\right) \qquad (7.2.7)$$

Where A is a real positive constant, ε and εA are small less than unity and v_0 is a scale of suction velocity which has non-zero positive constant.
In the free stream, from equation (7.2.2) we get

$$\rho \frac{d\bar{U}_\infty}{d\bar{t}} = -\frac{\partial \bar{p}}{\partial \bar{x}} - \rho_\infty g - \frac{\mu}{\bar{K}}\bar{U}_\infty - \sigma B_0^2 \bar{U}_\infty \qquad (7.2.8)$$

Eliminating $(\partial\bar{p}/\partial\bar{x})$ from equation (7.2.2) and equation (7.2.8), we obtain

$$\rho\left(\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = (\rho_\infty - \rho)g + \rho\frac{d\bar{U}_\infty}{d\bar{t}} + \mu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \left(\sigma B_0^2 + \frac{\mu}{\bar{K}}\right)(\bar{U}_\infty - \bar{u}) \quad (7.2.9)$$

Using the equation of state Hassanien and Obied Allah (2002)

$$\rho_\infty - \rho = \rho\beta(\bar{T} - \bar{T}_\infty) + \rho\bar{\beta}(\bar{C} - \bar{C}_\infty)\,, \qquad (7.2.10)$$

Where β is the volumetric coefficient of thermal expansion, $\bar{\beta}$ the volumetric coefficient of expansion with concentration and ρ_∞ the density of the fluid far away the surface and using equation (7.2.10) in (7.2.9) we get,

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \left\{ \begin{array}{c} \frac{d\bar{U}_\infty}{d\bar{t}} + \mu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \rho\beta(\bar{T} - \bar{T}_\infty) \\ +\rho\bar{\beta}(\bar{C} - \bar{C}_\infty) + \left(\frac{\sigma B_0^2}{\rho} + \frac{\nu}{\bar{K}}\right)(\bar{U}_\infty - \bar{u}) \end{array} \right\}, \tag{7.2.11}$$

where $\nu = \mu/\rho$ is the coefficient of the kinematic viscosity. The third term on the RHS of the momentum equation (7.2.11) denote body force due to non-uniform temperature, the fourth term denote body force due to non-uniform concentration.

The radiative heat flux term by using the Roseland approximation is given by

$$\bar{q}_r = \frac{4\bar{\sigma}}{3\bar{a}}\frac{\partial \bar{T}^4}{\partial \bar{y}}, \tag{7.2.12}$$

Where $\bar{\sigma}$ and $\bar{a}$ are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. We assume that the temperature differences within the flow are sufficiently small such that $\bar{T}^4$ may be expressed as linear function of the temperature. This is accomplished by expanding in a Taylor series about $\bar{T}_\infty$ and neglecting higher order terms, thus

$$\bar{T}^4 \cong 4\bar{T}_\infty^3 - 3\bar{T}_\infty^4 \tag{7.2.13}$$

With the help of equations (7.2.12) and (7.2.13), the equation (7.2.3) is reduced to

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{\kappa}{\rho C_p}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - \frac{16\bar{\sigma}\bar{T}_\infty^3}{3\rho C_p \bar{a}}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - \frac{Q_0}{\rho C_p}(\bar{T} - \bar{T}_\infty) \tag{7.2.14}$$

The non-dimensional quantities and parameters are

$$\left\{\begin{array}{c} \bar{u} = uU_0,\ \ \bar{v} = vV_0,\ \ \bar{U} = U_\infty U_0,\ \ \bar{u}_p =\ U_pU_0\ ,\ \ y = \dfrac{V_0\bar{y}}{\nu}, \\ \bar{T} = \bar{T}_\infty + \theta(\bar{T}_w - \bar{T}_\infty),\ \ \bar{C} = \bar{C}_\infty + \Phi(\bar{C}_w - \bar{C}_\infty), \\ \bar{K} = \dfrac{K}{V_0^2},\ \ Gr = \dfrac{\nu g\beta(\bar{T}_w - \bar{T}_\infty)}{U_0V_0^2}\ ,\ \ Gm = \dfrac{\nu g\bar{\beta}(\bar{C}_w - \bar{C}_\infty)}{U_0V_0^2}, \\ t = \dfrac{\bar{t}V_0^2}{\nu},\ \ \bar{n} = \dfrac{V_0^2}{\nu},\ \ Pr = \dfrac{\nu\rho C_p}{\kappa} = \dfrac{\nu}{\alpha}, \\ M = \dfrac{\sigma B_0^2\nu}{\rho V_0^2}\ ,\ \ R = \dfrac{4\bar{\sigma}\bar{T}_\infty^3}{\bar{a}\kappa}\ ,\ \ Sc = \dfrac{\nu}{D}\ ,\ \ Q = \dfrac{Q_0\nu}{\rho C_pV_0^2}\ , \end{array}\right\} \tag{7.2.15}$$

Taking into account the equation (7.2.7) and with the help of (7.2.15) the non-dimensional form of the equation (7.2.11), (7.2.14), (7.2.4) are as follows

$$\frac{\partial u}{\partial t} - (1 + \varepsilon Ae^{nt})\frac{\partial u}{\partial y} = \frac{dU_\infty}{dt} + N(U_\infty - u) + \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\phi\ , \tag{7.2.16}$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon Ae^{nt})\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\left(1 + \frac{4R}{3}\right)\frac{\partial^2 \theta}{\partial y^2} - Q\theta\ , \tag{7.2.17}$$

$$\frac{\partial \phi}{\partial t} - (1 + \varepsilon Ae^{nt})\frac{\partial \phi}{\partial y} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} + S_0\frac{\partial^2 \theta}{\partial y^2}\ , \tag{7.2.18}$$

where $N = M + K^{-1}$, *Gr* is the thermal Grashoff number, *Gm* is the solutal Grashoff number, *Pr* is the Prandtl number, *M* is the magnetic field parameter, *Sc* is the Schmidt number, *Q* is the dimensionless heat generation /absorption parameter, S_0 is the Soret number and *R* is the radiation parameter.

The corresponding non-dimensional boundary conditions are

$$\left\{\begin{array}{c} u = U_p\ ,\ \ \theta = 1 + \varepsilon e^{nt},\ \ \phi =\ 1 + \varepsilon e^{nt} \quad at\ y = 0 \\ u \to U_\infty = 1 + \varepsilon e^{nt},\ \ \theta \to 0,\ \ \phi \to 0 \quad as\ y \to \infty \end{array}\right\}, \tag{7.2.19}$$

7. 3 Method of Solution:

The equations (7.2.16)-(7.2.18) represent a set of partial differential equations and thus in order to reduce these into a set of ordinary differential equations in dimensionless form we assume the following for velocity, temperature and concentration as,

$$\left\{\begin{array}{l} u = u_0(y) + \varepsilon e^{nt} u_1(y) + 0(\varepsilon^2) \\ \theta = \theta_0(y) + \varepsilon e^{nt} \theta_1(y) + 0(\varepsilon^2) \\ \phi = \phi_0(\mathrm{y}) + \varepsilon \mathrm{e}^{\mathrm{nt}} \phi_1(\mathrm{y}) + 0(\varepsilon^2) \end{array}\right\}, \tag{7.3.1}$$

Where u_0 , θ_0 and ϕ_0 are mean velocity, mean temperature and mean concentration respectively.

Substituting the equation (7.3.1) into equations (7.2.16)-(7.2.18), equating the harmonic and non-harmonic terms and neglecting the higher-order terms of $0(\varepsilon^2)$, we obtain the following pairs of equations for (u_0 , θ_0 ϕ_0) and (u_1, θ_1, ϕ_1).

$$u_0'' + u_0' - Nu_0 = -N - Gr\theta_0 - Gm\phi_0 , \tag{7.3.2}$$

$$u_1'' + u_1' - (N + n)u_1 = -(N + n) - Au_0' - Gr\theta_1 - Gm\phi_1, \tag{7.3.3}$$

$$(3 + 4R)\theta_0'' + 3Pr\theta_0' - 3QPr\theta_0 = 0 , \tag{7.3.4}$$

$$(3 + 4R)\theta_1'' + 3Pr\theta_1' - 3Pr(Q + n)\theta_1 = -3APr\theta_0' , \tag{7.3.5}$$

$$\phi_0'' + Sc\ \phi_0' = -Sc\ S_0\theta_0'' , \tag{7.3.6}$$

$$\phi_1'' + Sc\ \phi_1' - Scn\ \phi_1 = -ASc\ \phi_0' - Sc\ S_0\theta_1'' , \tag{7.3.7}$$

The corresponding boundary conditions are

$$\left\{\begin{array}{l} u_0 = u_p ,\quad u_1 = 0,\quad \theta_0 = 1 ,\ \ \theta_1 = 1 ,\ \ \phi_0 = 1 ,\ \ \phi_1 = 1 \quad at\ \ y = 0 \\ u_0 \to 1 ,\ u_1 \to 1,\ \theta_0 \to 0 ,\theta_1 \to 0 ,\phi_0 \to 0 ,\phi_1 \to 0 \quad as\ \ y \to \infty \end{array}\right\}, \tag{7.3.8}$$

The solutions of equations (7.3.2) to (7.3.7) with the help of boundary conditions (7.3.8) are obtained as follows:

$$u_0 = 1 + A_1 e^{\beta_2 y} + A_2 e^{\beta_6 y} + A_3 e^{\beta_2 y} + A_4 e^{\beta_{10} y}, \tag{7.3.8}$$

$$u_1 = \left\{ \begin{matrix} A_6 e^{\beta_{10} y} + A_7 e^{\beta_2 y} + A_8 e^{\beta_6 y} + A_9 e^{\beta_2 y} + A_{10} e^{\beta_4 y} \\ + A_{11} e^{\beta_2 y} + A_{12} e^{\beta_8 y} + A_{13} e^{\beta_6 y} + A_{14} e^{\beta_2 y} + A_{15} e^{\beta_{12} y} \end{matrix} \right\}, \tag{7.3.9}$$

$$\theta_0 = e^{\beta_2 y}, \tag{7.3.10}$$

$$\theta_1 = L_1 e^{\beta_2 y} + L_2 e^{\beta_4 y}, \tag{7.3.11}$$

$$\phi_0 = P_1 e^{\beta_2 y} + P_2 e^{\beta_6 y}, \tag{7.3.12}$$

$$\phi_1 = P_3 e^{\beta_6 y} + P_4 e^{\beta_2 y} + P_5 e^{\beta_8 y}, \tag{7.3.13}$$

Thus the expression for the velocity, temperature and concentration profiles are as follows

$$u(y,t) = \left\{ \begin{matrix} 1 + A_1 e^{\beta_2 y} + A_2 e^{\beta_6 y} + A_3 e^{\beta_2 y} + A_4 e^{\beta_{10} y} \\ + \varepsilon e^{nt} (A_6 e^{\beta_{10} y} + A_7 e^{\beta_2 y} + A_8 e^{\beta_6 y} + A_9 e^{\beta_2 y} + A_{10} e^{\beta_4 y} \\ + A_{11} e^{\beta_2 y} + A_{12} e^{\beta_8 y} + A_{13} e^{\beta_6 y} + A_{14} e^{\beta_2 y} + A_{15} e^{\beta_{12} y}) \end{matrix} \right\}, \tag{7.3.14}$$

$$\theta(y,t) = e^{\beta_2 y} + \varepsilon e^{nt} \left(L_1 e^{\beta_2 y} + L_2 e^{\beta_4 y} \right), \tag{7.3.15}$$

$$\phi(y,t) = P_1 e^{\beta_2 y} + P_2 e^{\beta_6 y} + \varepsilon e^{nt} \left(P_3 e^{\beta_6 y} + P_4 e^{\beta_2 y} + P_5 e^{\beta_8 y} \right). \tag{7.3.16}$$

The physical quantities of interest are the wall shear stress is given by

$$\bar{\tau}_w = \mu \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)_{\bar{y}=0},$$

And in dimensionless form we get,

$$C_f = \frac{\bar{\tau}_w}{\rho U_0 V_0} = \left(\frac{\partial u}{\partial y} \right)_{y=0} = u'(0)$$

$$=\begin{Bmatrix} A_1\beta_2 + A_2\beta_6 + A_3\beta_2 + A_4\beta_{10} + A_6\beta_{10} + A_7\beta_2 + A_8\beta_6 \\ +A_9\beta_2 + A_{10}\,\beta_4 + A_{11}\beta_2 + A_{12}\beta_8 + A_{13}\beta_6 + A_{14}\beta_2 + A_{15}\beta_{12} \end{Bmatrix} \qquad (7.3.17)$$

The local surface heat flux is given by

$$\bar{q}_w = -\kappa\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=0} = -\frac{4\bar{\sigma}}{3\bar{a}}\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=0}$$

Using equation (7.2.13) the above equation can be written as

$$\bar{q}_w = -\kappa\left(\kappa + \frac{16\bar{\sigma}\bar{T}_\infty}{3\bar{a}}\right)\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=0},$$

and its non-dimensional form is

$$\bar{q}_w = -\frac{\kappa(\bar{T}_w - \bar{T}_\infty)V_0}{\nu}\left(1 + \frac{4R}{3}\right)\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

The dimensionless local surface heat flux that is Nusselt number is obtained as

$$Nu_x = \frac{\bar{q}_w}{\kappa(\bar{T}_w - \bar{T}_\infty)}$$

$$\therefore \frac{Nu_x}{Re_x} = -\left(1 + \frac{4R}{3}\right)\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = -\left(1 + \frac{4R}{3}\right)(\beta_2 + \beta_2 L_1 + \beta_4 L_2), \qquad (7.3.18)$$

Where, $Re_x = V_0 x/\nu$ is the local Renolds number.

The definition of the local mass flux and the local Sherwood number are given by

$$j_w = -D\left(\frac{\partial \bar{C}}{\partial \bar{y}}\right)_{\bar{y}=0}$$

$$Sh_x = \frac{j_w x}{D(\bar{C}_w - \bar{C}_\infty)}$$

$$\therefore \frac{Sh_x}{Re_x} = -\left(\frac{\partial \Phi}{\partial y}\right)_{y=0} = P_1\beta_2 + P_2\beta_6 + P_3\beta_6 + P_4\beta_2 + P_5\beta_8 \qquad (7.3.19)$$

7.4 Validity:

In order to verify the accuracy of the present results, we have considered the analytical solutions obtained by Kim (2000) for local Nusselt number. These compared results are presented in the Table 7.4(a). It is observe form this Table that the present results (under some limiting conditions) are in very good agreement with those obtained from analytical solutions of Kim (2000), which clearly shows the correctness of the present analytical solutions and the available solutions in the literature.

From the Table – 7.4(a), it is seen that a negative increasing has been occurred in Nu_x/Re_x when either Pr or F is increased.

Table 7.4(a) Comparison of analytical results with those of Kim (2000) with different values of Pr for Nu_x/Re_x when $R = 0, t = 0.2, \varepsilon = 0.001, n = 0.2, A = 0.05$, $Q = 0, S_0 = 0, Sc = 0.6$:

Pr	**Kim (2000) results**	**Present results**
	Nu_x/Re_x	Nu_x/Re_x
0.71	− 0.353081	− 0.353077
1.0	− 0.673049	− 0.673107
7.0	− 1.737483	− 1.737490
11.4	− 3.494173	− 3.494183

7.5 Results and Discussion:

In this section, the effects of various physical parameters like magnetic body force, porosity, heat generation, thermal radiation and Soret number have been discussed on the flow velocity, temperature, concentration, skin friction and Nusselt number. All the numerical calculations are done with respect to air (*Pr* = 0.71) at 20oC and steam (Sc = 0.60).

Fig. 7.4 (i), analyses that the velocity distribution has been decreased due to the effect of magnetic drag force (M), which resists the motion of the flow due to Lorentz force.

The effect of heat generation (*Q*) on the flow velocity and temperature are presented in Figs. 7.4 (ii) and 7.4 (iii) respectively. Both the momentum and thermal boundary layers are reduced for the effect of heat generation and consequently the velocity and temperature profiles are decreased by the increasing values of heat generation parameter.

In Fig. 7.4 (iv), it is observed that an increase in radiation (*R*) leads to decrease the flow velocity.

In Fig. 7.4 (v), the porosity of the medium has increased the flow velocity in the momentum boundary layer.

The Soret (S_0) effect on the velocity, concentration and Sherwood number have been plotted in the respective Figs. 7.4 (vi), 7.4 (vii) and 7.4 (viii). The velocity has been elevated for the increasing values of S_0 and attain its peak values near the plate (y = 0). The similar effect is also observed for concentration profiles, but the graphs of the concentration have been depressed sharply for greater values of the distance (y). Moreover, the negative increasing effect (decreased in magnitude) for the Soret number has been observed in the Sherwood number, whereas Sherwood number is substantially increased when the Schmidt number is gradually increased.

In Fig. 7.4 (ix), the Skin friction (τ) at y = 0 for Q and S_0 is plotted. Due to Soret effect, the skin friction has been increased near the plate y = 0, but a reverse trend has been observed away the plate. However, τ has a negative depression for Q∈[0, 3] and has a positive escalation for Q∈[3, 6].

Nusselt number (Nu) for *R* and *Q* is displayed in Fig. 7.4 (x). Due to heat generation, the Nusselt number is substantially increased in presence of thermal radiation.

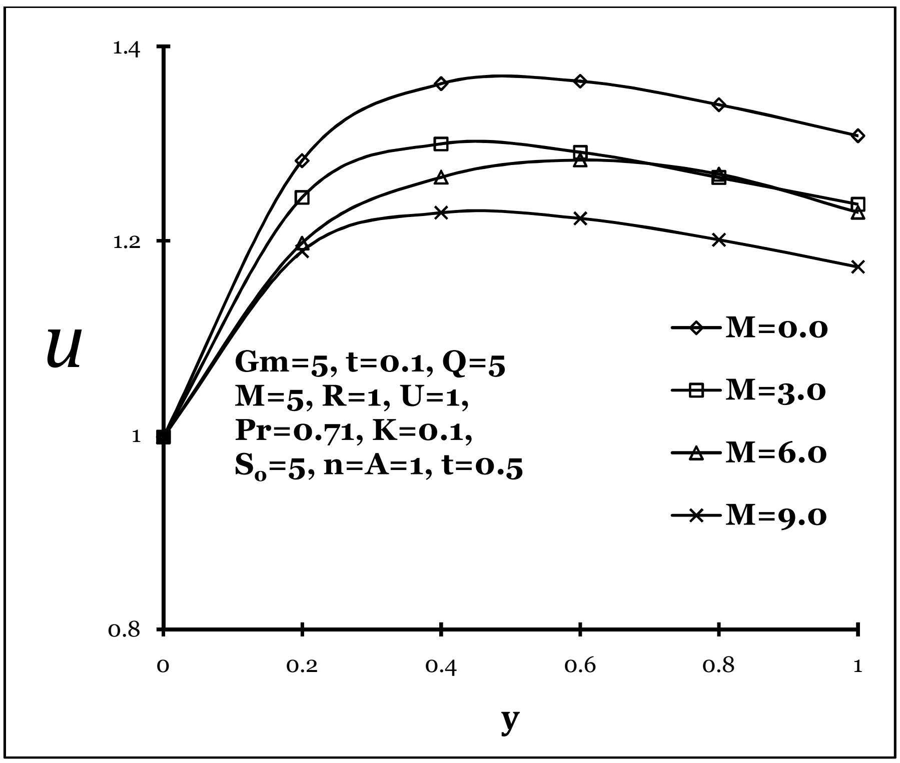

Fig. 7.4 (i): Velocity distribution on *M*

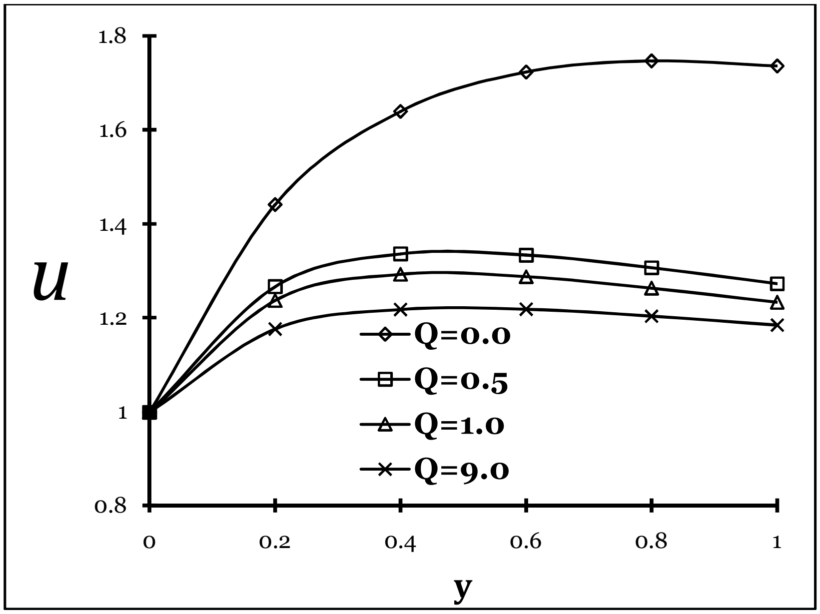

Fig. 7.4 (ii): Velocity distribution on *Q*

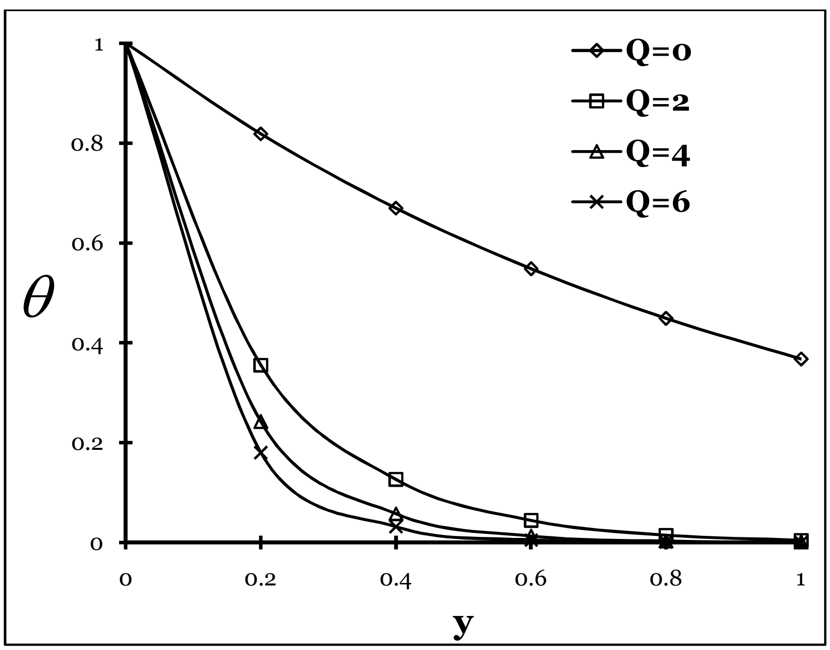

Fig. 7.4 (iii): Temperature for *Q*

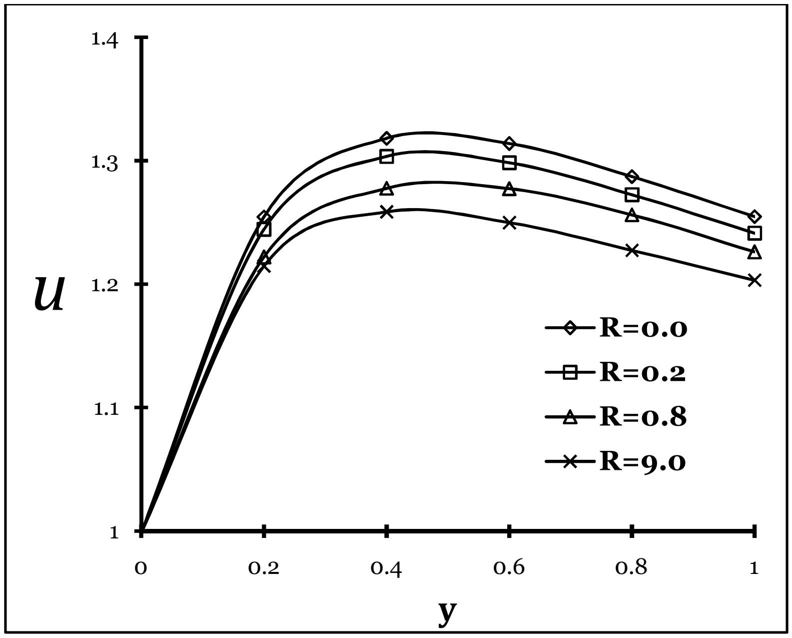

Fig. 7.4 (iv): Velocity distribution on *R*

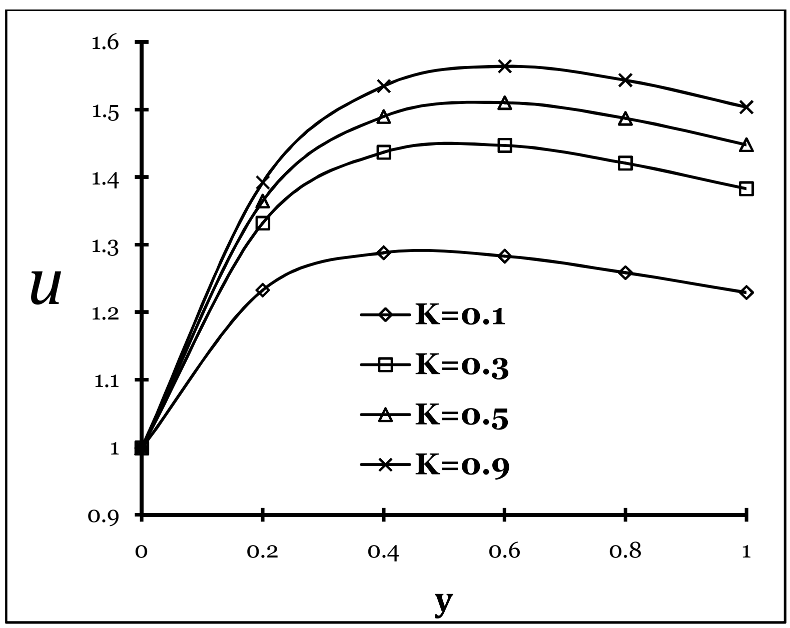

Fig. 7.4 (v): Velocity distribution on *K*

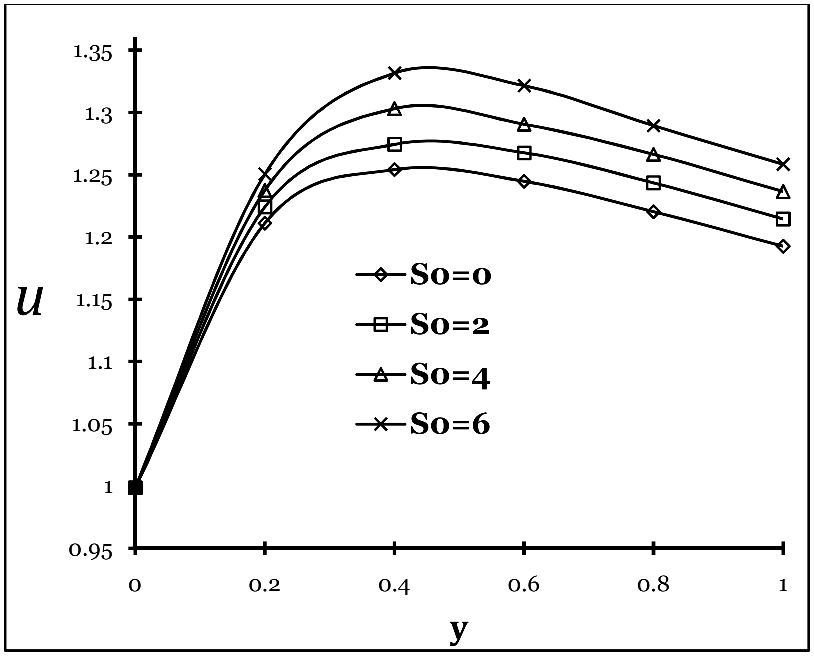

Fig. 7.4 (vi): Velocity distribution for S_o

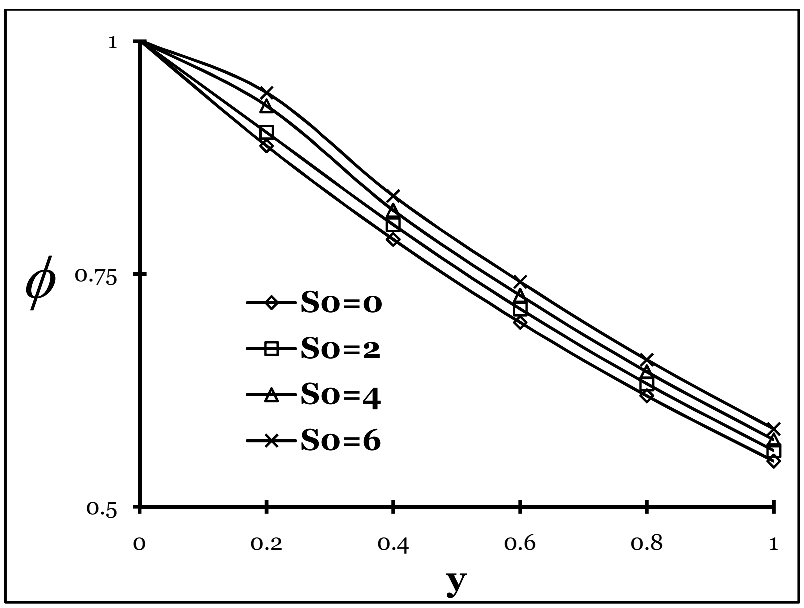

Fig. 7.4 (vii): Concentration for ∅

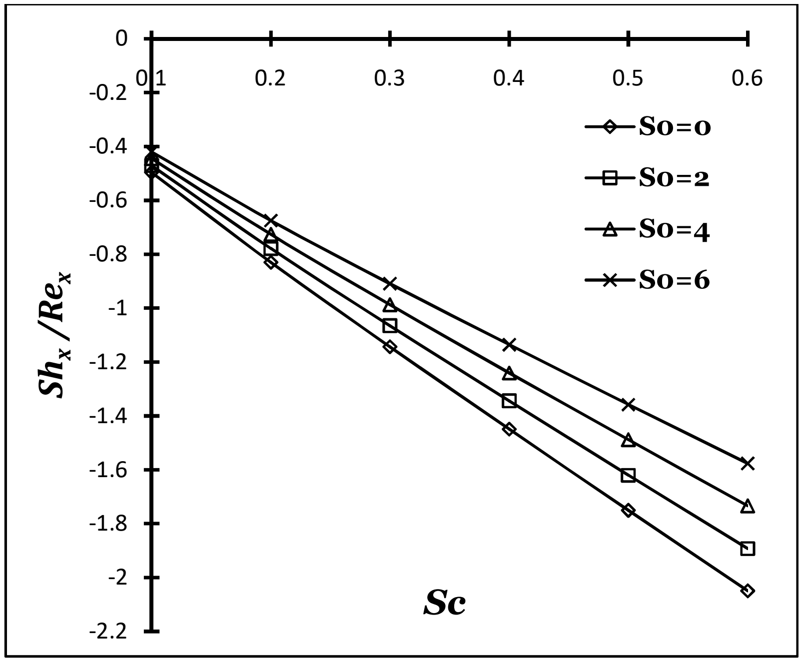

Fig. 7.4 (viii): Sherwood number for *Sc* and S_0

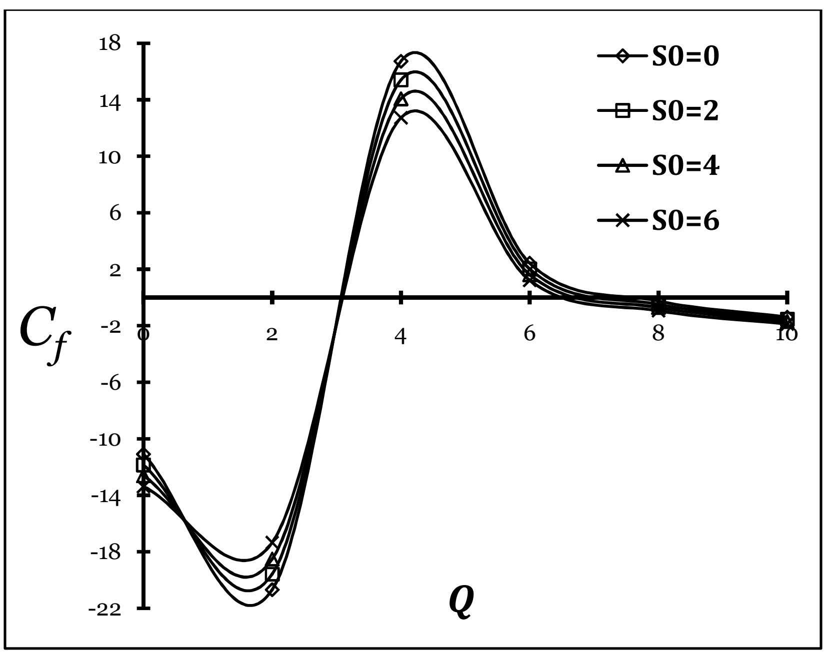

Fig. 7.4 (ix): Skin friction for *Q* and S_0

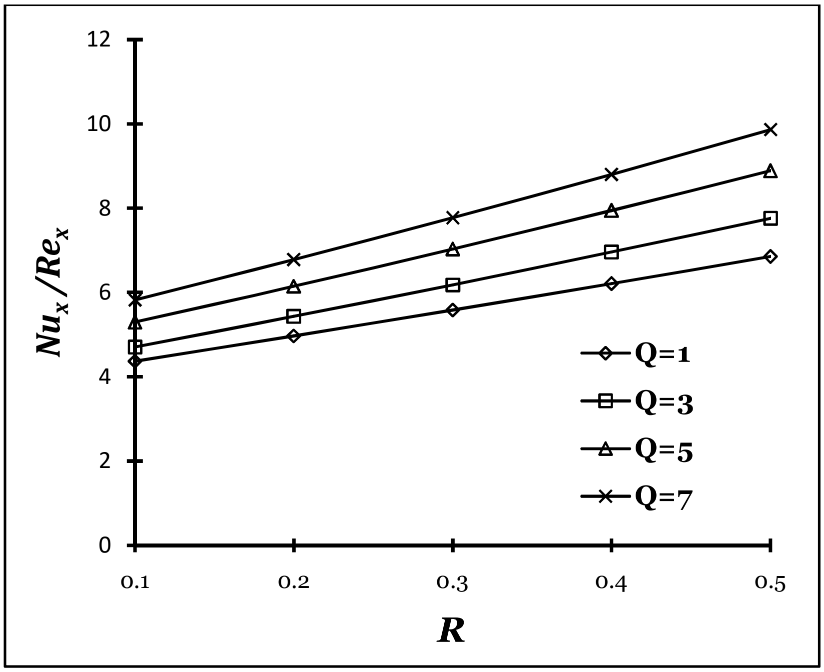

Fig. 7.4 (x): Nusselt number for *R* and *Q*

7.6 Conclusions:

The present study brings out the following significant findings:

- The heat generation has a decelerating effect on the flow velocity. This decreases the temperature as well. But this heat generation raises the temperature gradients.
- The porosity parameter, the more sharply is the elevation in velocity.
- Transverse magnetic field produces a type of resistive force which opposes the flow. This contributes to the thickening of the thermal and mass boundary layer which in turn, reduces the rate of heat and mass transfer.
- The increase of Soret effect is seen to accelerate the flow velocity, species concentration as well as the concentration gradients.