Write the following information in the first page of Answer Script before starting answer

ODD SEMESTER EXAMINATION: 2020-21

Exam ID Number______________________________

Course____________________________ Semester____________________________

Paper Code _________________Paper Title __________________________________

Type of Exam: ______________________________ (Regular/Back/Improvement)

Important Instruction for students:

1. Student should write objective and descriptive answer on plain white paper.
2. Give page number in each page starting from 1st page.
3. After completion of examination, Scan all pages, convert into a single PDF, rename the file with Class Roll No. **(2019MBA15)** and upload to the Google classroom as attachment.
4. Exam timing from 10am – 1pm (for morning shift).
5. Question Paper will be uploaded before 10 mins from the schedule time.
6. Additional 20 mins time will be given for scanning and uploading the single PDF file.
7. Student will be marked as ABSENT if failed to upload the PDF answer script due to any reason.

M.Sc. MATHEMATICS
THIRD SEMESTER
NUMBER THEORY
MSM-301

Duration : 3 hrs. **Full Marks : 70**

(PART-A : Objective)

Time : 20 min. **Marks : 20**

Choose the correct answer from the following: ***1X20=20***

1. Which of the following(s) is/are not perfect number?
   - **a.** 496
   - **b.** 8128
   - **c.** Both (a) and (b)
   - **d.** None of these

2. The remainder of $4(29!) + 5!$ divided by 31 is:
   - **a.** 00
   - **b.** 01
   - **c.** 02
   - **d.** None of these

3. The value of $\tau(180)$ and $\sigma(180)$ are respectively:
   - **a.** 17 & 546
   - **b.** 546 & 17
   - **c.** 546 & 18
   - **d.** 18 & 546

4. If a is a quadratic non-residue modulo an odd prime p then:
   - **a.** $a^{\frac{p-1}{2}} \equiv 1 (\text{mod } p)$
   - **b.** $a^{\frac{p-1}{2}} \equiv -1 (\text{mod } p)$
   - **c.** $a^{\frac{p-1}{2}} \equiv \pm 1 (\text{mod } p)$
   - **d.** All of these

5. If $2^k - 1$ is prime then:
   - **a.** k is composite
   - **b.** k is prime
   - **c.** k is any integer
   - **d.** Such k does not exist

6. Which of the following statement(s) is/are necessarily true?
   - **a.** $\phi(n) \mid n$ for all positive integers n
   - **b.** $n \mid \phi(a^n - 1)$for all positive integers a & n
   - **c.** $n \mid \phi(a^n - 1)$ for all positive integers a & n such that $\gcd(a, n) = 1$
   - **d.** $a \mid \phi(a^n - 1)$for all positive integers a & n such that $\gcd(a, n) = 1$

7. The continued fraction of $\frac{118}{303}$ is:
   - **a.** $[0;\ 2, 1, 1, 3, 5, 3]$
   - **b.** $[0, ; 2, 1, 3, 5, 3]$
   - **c.** $[\ 2; 1, 3, 5, 3]$
   - **d.** $[\ 2;\ 1, 1, 3, 5, 3]$

8. The congruence $6x \equiv 1\ (\text{mod } 9)$ has:
   - **a.** 3 solutions
   - **b.** At least 3 solution
   - **c.** Exactly 3 solutions
   - **d.** No solution

9. If $\frac{p}{q}$ is convergent of $\sqrt{d}$ then:
   - **a.** $\left|\sqrt{d} - \frac{p}{q}\right| < \frac{1}{p^2}$
   - **b.** $\left|\sqrt{d} - \frac{p}{q}\right| > \frac{1}{p^2}$

c. $\left|\sqrt{d}-\frac{p}{q}\right|<\frac{1}{q^2}$

d. $\left|\sqrt{d}-\frac{p}{q}\right|>\frac{1}{q^2}$

10. The congruence $x^2 \equiv a \pmod{32}$ has a solution for which of the value of a?

a. 9
b. 13
c. 15
d. None of these

11. Find the last two digits of the number 3^{256}.

a. 20
b. 21
c. 22
d. None of these

12. For any odd prime p, $\sum_{a=1}^{p-1}\left(\frac{a}{p}\right)$ is equal to:

a. -1
b. 0
c. 1
d. Given information is not sufficient

13. The number of primitive roots of 343 is:

a. 294
b. 342
c. 84
d. 42

14. Suppose ϕ and φ denotes Golden ratio and Golden ratio conjugate respectively. The nth Fibonacci number F_n is equal to:

a. $F_n = \frac{\phi^n - \varphi^n}{\sqrt{5}}$

b. $F_n = \frac{\phi^n - (-\varphi)^n}{\sqrt{5}}$

c. $F_n = \frac{(\phi)^n + (-\varphi)^n}{\sqrt{5}}$

d. $F_n = \frac{(\phi)^n + \varphi^n}{\sqrt{5}}$

15. For any prime $p > 5$, $\gcd(F_{p-1}, F_{p+1})$ is:

a. Always 1
b. Any positive integer
c. Not always 1
d. None of these

16. The sequence $C_1, C_3, C_5, \cdots$ is:

a. Decreasing sequence
b. Strictly decreasing sequence
c. Increasing sequence
d. Strictly increasing sequence

17. If p_n denotes the nth prime number then which of the following is/are true?

a. $p_n \leq p_1 p_2 \cdots p_{n-1} - 1 \quad n \geq 2$
b. $p_n \leq p_1 p_2 \cdots p_{n-1} \quad n \geq 2$
c. $p_n \leq p_1 p_2 \cdots p_{n-1} + 1 \quad n \geq 2$
d. None of these

18. Which of the following number(s) has no primitive root?

a. 243
b. 250
c. 256
d. None of these

19. The value of $F_{n+2}^2 - F_n^2$ is:

a. F_{2n-1}
b. F_{2n}
c. F_{2n+1}
d. F_{2n+2}

20. If n is a perfect number then:

$\sum_{d|n} \frac{1}{d}$ is:

a. 1
b. 2
c. A positive integer other than 1 and 2
d. A real number other than 1 and 2

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PART-B : Descriptive

Time : 2 hrs. 40 min. **Marks : 50**

[Answer question no.1 & any four (4) from the rest]

1. **a)** Find all the solutions of the following system of linear congruences in the interval $[801, 1000]$ 4+4+2=10

$x \equiv 5 (\text{mod } 6)$
$x \equiv 4 (\text{mod } 11)$
$x \equiv 3 (\text{mod } 7)$

b) Determine all solutions in the positive integers of $54x + 21y = 906$.

c) For an odd prime p, prove that the congruence $2x^2 + 1 \equiv 0 \ (\text{mod } p)$ has a solution if and only if $p \equiv 1 \text{ or } 3 \ (\text{mod } 8)$.

2. **a)** Solve the following linear congruence $140x \equiv 133 \ (\text{mod } 301)$. 4+3+3=10

b) What is the remainder when the following sum is divisible by 4?
$1^5 + 2^5 + 3^5 + \cdots + 99^5 + 100^5$

c) Prove that $17 \mid (11^{104} + 1)$.

3. **a)** Prove that - $\phi(2^n - 1)$ is a multiple of n for any $n > 1$. 5+3+2=10

b) Verify that 3 is a primitive root of $F_n, n > 1$.

c) Prove that -$2^{35} - 1$ is divisible by 127.

4. **a)** Find the solution of the following Pell's equation 5+5=10
$x^2 - 7y^2 = 1$

b) Show that the sum of the squares of the first n Fibonacci numbers is
$F_1^2 + F_2^2 + F_3^2 + \cdots + F_n^2 = F_n F_{n+1}$

5. **a)** Evaluate $[1; 1, 1, \cdots]$. 5+5=10

b) Solve the following congruence
$3x^4 \equiv 5 (\text{mod } 11)$

6. **a)** State and Prove Euler's criterion. 5+3+2=10

b) Determine whether the following congruence has solution or not:
$x^2 \equiv -46 \ (\text{mod } 17)$

c) Find the value of $\left(\frac{-23}{59}\right)$.

7. **a)** If p is a prime number and $d \mid (p-1)$, then the congruence 5+2+3=10
$x^d - 1 \equiv 0 \ (\text{mod } p)$
has exactly d solutions.

b) Prove that -The polynomial $f(n) = n^2 + n + 41$ is composite.

c) Prove that - If p_n is the nth prime number, then $p_n \leq 2^{2^{n-1}}$.

8. **a)** Prove that $L_n = F_{n-1} + F_{n+1}$. 4+4+2=10

b) Prove that – If p is an odd prime number and $k \geq 1$, then there exists a primitive root for p^k.

c) If p is an odd prime, then prove that

$$\left(\frac{-2}{p}\right) = \begin{cases} -1, & \text{if } p \equiv 5 (\text{mod } 8) \text{or } \ p \equiv 7 (\text{mod } 8) \\ 1, & \text{if } p \equiv 1 (\text{mod } 8) \text{or } \ p \equiv 3 (\text{mod } 8) \end{cases}$$

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