REV-01
MSM/58/66

2022/07

M.Sc. MATHEMATICS
SECOND SEMESTER
DIFFERENTIAL EQUATIONS-II
MSM-202

Duration : 3 hrs. **Full Marks: 70**

(PART-A: Objective)

Time : 20 min. **Marks : 20**

Choose the correct answer from the following: ***1X20=20***

1. For the Boundary value problem $\frac{d^2y}{dy^2} + \lambda y = 0$, $y(-\pi) = y(\pi), \frac{dy}{dx}(-\pi) = \frac{dy}{dx}(\pi)$ to each Eigen values λ there corresponds
   a. Only one Eigen value
   b. Two Eigen values
   c. Two linearly independent Eigen functions
   d. Two orthogonal Eigen functions

   a. a, c **b.** b, d
   c. c, d **d.** a, c, d

2. The Differential equation whose linearly independent solutions are $Cos\ 2x$, $\sin 2x$ and e^{-x} is

   a. $(D^3 + D^2 + 4D + 4)Y = 0$ **b.** $(D^3 - D^2 + 4D - 4)Y = 0$
   c. $(D^3 + D^2 + 4D - 4)Y = 0$ **d.** $(D^3 + D^2 - 4D + 4)Y = 0$

3. The differential equation of the family of circles of radius "r' whose center lie the x-axis, is

   a. $y\left(\frac{dy}{dx}\right) + y^2 = r^2$ **b.** $y\{\left(\frac{dy}{dx}\right) + 1\} = r^2$
   c. $y^2\{\left(\frac{dy}{dx}\right) + 1\} = r^2$ **d.** $y^2\{\left(\frac{dy}{dx}\right)^2 + 1\} = r^2$

4. The equation of the curve, of which the angle between the tangent and the radius vector is twice the vectorial angle is $r^2 = Asin\ 2\theta$. This satisfies the differential equation.

   a. $r(\frac{dr}{d\theta}) = \tan 2\theta$ **b.** $r(\frac{d\theta}{dr}) = \tan 2\theta$
   c. $r(\frac{dr}{d\theta}) = \cos 2\theta$ **d.** $r(\frac{d\theta}{dr}) = \cos 2\theta$

5. The maximum number of linearly independent solutions of the differential equations $\frac{d^4y}{dx^4} = 0$ with the condition $y(0) = 1$

   a. 4 **b.** 3
   c. 2 **d.** 1

6. Linear combinations of solutions of an ordinary differential equation are solutions the differential equation is

a. Linear non- homogeneous
b. Linear homogeneous.
c. Non -linear homogeneous
d. Non-linear non homogeneous

7. Let $y = \varphi(x)$and $y = \psi(x)$ be solutions of $y'' - 2xy' + (\sin x^2)y = 0$ such that $\varphi(x) = 0$, $\phi'(0) = 1$ and $\varphi(0) = 1$, $\varphi'(0) = 2$. The value of wromhian $W(\theta, \varphi)$ at $x = 1$is

a. 0
b. 1
c. e
d. e^2

8. What are the order and degree respectively of the differential equation

$\frac{d^2}{dx^2}\left(\frac{d^2y}{dx^2}\right)^{\frac{-3}{2}} = 0$

a. 1,4
b. 4,1
c. 4, 4
d. 1, 1

9. Determine the type of the following differential equation $\frac{d^2y}{dx^2} + \sin(x+y) = \sin x$

a. Linear, homogeneous
b. Non-linear, homogeneous
c. Linear, non- homogeneous
d. Non -linear, non- homogeneous

10. The solution of $(x - y^2)dx + 2xydy = 0$ is

a. $ye^{\frac{y^2}{x}=A}$
b. $xe^{\frac{y^2}{x}=A}$
c. $ye^{\frac{x}{y^2}=A}$
d. $xe^{\frac{x}{y^2}=A}$

11. The solution of $\frac{dy}{dx} + y\frac{d\varphi}{dx} = \varphi(x)\frac{d\varphi}{dx}$ is

a. $y = \varphi(x) - 1 + ce^{-\varphi}$
b. $y = ce^{\varphi}$
c. $y = x\varphi(x) - ce^{-\varphi}$
d. $y = [\varphi(x) - 1]e^{-\varphi} + c$

12. Which of the following is not an integrating factor of $xdy - ydx = 0$

a. $\frac{1}{x^2}$
b. $\frac{1}{x^2 + y^2}$

c. $\frac{1}{xy}$ d. $\frac{x}{y}$

13. The general solution of $\frac{dy}{dx} + \tan y \tan x = \cos x$ is

a. $2\sin y = (x + c - \sin x \cos x)\sec x$ b. $\sin y = (x + c)\cos x$

c. $Cos\, y = (x + c)\sin x$ d. $\sec y = (x + c)\cos x$

14. If the integrating factor of $(x^7y^2 + 3y)dx + (3x^8y - x)dy = 0$ is $x^m y^n$ then

a. $m = -7, n = 1$ b. $m = 1, n = -7$

c. $m = n = 0$ d. $m = n = 1$

15. The solution of $\frac{d^2s}{dt^2} = g$ (g is a constant, $s = 0$) and $\frac{ds}{dt} = u\,; t - 0$is

a. $s = gt$ b. $s = ut + \frac{1}{2}gt^2$

c. $s = \frac{1}{2}gt^2$ d. None

16. The solution of $\frac{d^2y}{dx^2} - y = k$, (K a non-zero constant) which vanishes when x=0 and which tends to finite limit as x tends to infinite as

a. $y = k(1 + e^{-x})$ b. $y = k(e^x + e^{-x} - 2)$

c. $y = k(e^{-x} - 1)$ d. $y = k(e^x - 1)$

17. If $e^{ax}u(x)$ is a particular integral $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + a^2y = f(x)$; where a is a constant then $\frac{d^2u}{dx^2}$ is equal to

a. $f(x)$ b. $f(x)e^x$

c. $f(x)e^{-ax}$ d. $f(x)(e^{ax} + e^{-ax})$

18. Consider the equation of an ideal planar pendulum given by $\frac{d^2y}{dy^2} = -\sin x$; where x denotes the angle of displacement. For sufficiently small angels of displacement the solution is given by (Where a, b are constants)

a. $x(t) = a\cosh t + b\sinh t$ b. $x(t) = a + bt$

c. $x(t) = ae^t + be^{2t}$ d. $x(t) = a\cos t + b\sin t$

19. The boundary value problem $\frac{d^2y}{dx^2} + y = cosec\, x$, $0 \leq x \leq \frac{\pi}{2}$; $y(0) = 0,\ y\left(\frac{\pi}{2}\right) = 0$, is

a. convex
b. concave
c. negative
d. none

20. The Eigen value of the BVP $\frac{d^2y}{dy^2} + \lambda y = 0$, $x(0) = 0;\ x(\pi) + x'(\pi)) = 0$ satisfy

a. $\lambda + \tan \lambda\pi = 0$
b. $\sqrt{\lambda} + \tan \lambda\pi = 0$
c. $\sqrt{\lambda} + \tan \sqrt{\lambda}x = 0$
d. $\lambda + \tan \sqrt{\lambda}x = 0$

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(PART-B : Descriptive)

Time: 2 HRS 40 MINS Marks : 50

[Answer question no.(1) & any four (4) from the rest]

1. a. State the existence and uniqueness theorem for the nth order differential equation $L(y)(x) = y^n(x) + p_1(x)y^{n-1}(x) + \cdots \ldots \ldots \ldots + p_n(x)y(x) = 0, x \in I$, which is a linear homogeneous equation? 5+5=10

   b. Show that there are three independent solutions of the 3rd order equation

   $y'''(x) + p_1(x)y''(x) + p_2(x)y'(x) + p_3(x)y(x) = 0, x \in I$,

   Where $p_1(x)$, $p_2(x)$, and $p_3(x)$ are the functions defined continuous on an interval I.

2. a. Reduce the differential equation $(px - y)(py + x) = h^2p$ to Clairaut's form by substitution $x^2 = u, y^2 = v$ and find its complex primitive. 5+5=10

   b. Solve $D^2 - 2D + 1)y = x^2e^{2x}$

3. 5+5=10

   a. Solve $y'' - \frac{3}{x}y' + \frac{3}{x^2}y = 2x - 1$

   b. Solve $y'' - x^2y' + xy = x$

4. a. Define Strum-Liouville equation, Eigen function and Eigen value. 4+6=10

   b. Find the Eigen values and the corresponding Eigen functions of

   $X'' + \lambda X = 0,,, X(0) = 0.,, X'(L) = 0$

5. a. Define Linear Equation and semi linear equation. 4+6=10

   b. Find the PDE by eliminating arbitrary constant a and b from the following equations $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

6. Transformed the Laplace equation $\frac{\delta^2 v}{\delta x^2} + \frac{\delta^2 v}{\delta y^2} = 0$ into polar coordinate (r, θ) 10

7. a. Solve $r = a^2 t$ 5+5=10

b. Find the complete integral of $p_1^3 + p_2^3 + p_3^3 = 1$

8. a. Verify that one solution of the equation $(1 - x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ is $y = x$ and find another solution valid in the interval $-1 < x < 1$. 5+5=10

b. Solve $(D^2 - 9)y = 6e^{3x} + xe^{3x}$

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