

**M.Sc. MATHEMATICS**  
**THIRD SEMESTER (SPECIAL REPEAT)**  
**CONTINUUM MECHANICS & HYDRODYNAMICS**  
**MSM-304**

(Use separate answer scripts for Objective & Descriptive)

Duration : 3 hrs.

Full Marks : 70

( PART-A : Objective )

Time : 20 min.

Marks : 20

*Choose the correct answer from the following:*

*1X20=20*

- The equation  $|\sigma_{ij} - \delta_{ij}\sigma| = 0$  upon expansion gives a \_\_\_\_ polynomial.
  - Biquadratic
  - Quadratic
  - Quadric
  - None of these
- The principal directions of Deviator Stress tensor are the same as those of \_\_\_\_\_.
  - Unit vector
  - Stress tensor
  - Normal tensor
  - None of these
- Inertia force is an example of:
  - Surface force
  - Body force
  - Both of these
  - None of these
- A material having identical property at all points is \_\_\_\_ property.
  - Isophagus
  - Homogeneous
  - Eulerian
  - None of these
- A property which is directional at a point is known as \_\_\_\_\_.
  - Unhomogeneous
  - Anisotropic
  - Anisophagus
  - None of these
- In Continuum concept, the components perpendicular to the plane are \_\_\_\_\_.
  - Traction stresses
  - Shear stresses
  - Unit stresses
  - None of this
- We define \_\_\_\_\_ as the possible pairs of traction vector and unit normal.
  - Normal stress
  - State of stress
  - Stress principle
  - None of these
- The magnitude of normal stress component in quadric surface of Cauchy is \_\_\_\_\_ to  $r^2$ .
  - Inversely proportional
  - Equal
  - Directly proportional
  - None
- In the three equation of  $(\sigma_{ij} - \delta_{ij}\sigma)n_j = 0$ , there are \_\_\_\_\_ unknown.
  - Three
  - Six
  - Five
  - None of these

10. A necessary and sufficient condition for the inverse function to exist is Jacobian\_\_\_\_\_.
- Vanishes
  - Does not vanish
  - Both of them
  - None of them
11. Stress tensor is\_\_\_\_\_.
- Equivalent
  - Asymmetric
  - Symmetric
  - None of these
12.  $C_{ij} = \frac{\partial X_k}{\partial x_i} \frac{\partial X_k}{\partial x_j}$  is called the:
- Undeformed tensor
  - Green's deformation tensor
  - Cauchy's deformation tensor
  - None of these
13.  $G_{ij} = \frac{\partial x_k}{\partial X_i} \frac{\partial x_k}{\partial X_j}$  is called the:
- Undeformed tensor
  - Green's deformation tensor
  - Cauchy's deformation tensor
  - None of these
14. A divergence of a vector f is written as:
- $\nabla f$
  - $\nabla \cdot f$
  - $\nabla \times f$
  - None of these
15.  $\frac{1}{2} \left( \delta_{ij} - \frac{\partial X_k}{\partial x_i} \frac{\partial X_k}{\partial x_j} \right)$  is equal to:
- $\delta_{ij}$
  - $E_{ij}$
  - $L_{ij}$
  - None of these
16. A gradient of a vector f is written as:
- $\nabla f$
  - $\nabla \cdot f$
  - $\nabla \times f$
  - None of these
17. For small deformation theory we have:
- $l_{ij} = C_{ij}$
  - $l_{ij} \neq e_{ij}$
  - $l_{ij} = e_{ij}$
  - $l_{ij} \neq C_{ij}$
18. For incompressible flow, with fluid velocity q, the equation of continuity is:
- Curl. q is zero
  - Div. q is zero
  - Grad q is zero
  - 1
19. From the law of conservation of mass, the mass contained inside a given volume of fluid remains \_\_\_\_\_ throughout the motion.
- Changing
  - Changes from time to time
  - Unchanged
  - None of these
20. A flow where the particles move parallelly to each other is known as:
- Laminar
  - Turbulent
  - None of these
  - All of these

**( PART-B : Descriptive )**

Time : 2 hrs. 40 min.

Marks : 50

[ Answer question no.1 & any four (4) from the rest ]

1. Explain the continuum concept. Deduce the relationship between stress vector and stress tensor. 5+5=10
2. A displacement field is given by  $x_1 = X_1 + Ax_2$ ,  $x_2 = X_2 + Ax_3$ ,  $x_3 = X_3 + Ax_1$ . Calculate the Lagrangian linear strain tensor and Eulerian linear strain tensor. Compare them to the case when A is small. 5+5=10
3. The stress tensor at a point is given by  $\sigma_{ij} = \begin{pmatrix} 0 & 1 & 2 \\ 1 & \sigma_{22} & 1 \\ 2 & 1 & 0 \end{pmatrix}$ . 5+5=10  
Determine  $\sigma_{22}$  so that the stress vector on the same plane at the given point will be zero. Explain the stress quadric of Cauchy.
4. Explain the material and spatial methods of description of analysis of strain. Write about small deformation theory. 5+5=10
5. At a point in an incompressible fluid having spherical polar coordinates  $(r, \theta, \varphi)$ , the velocity components are given by  $2Mr^{-3} \cos\theta$ ,  $Mr^{-3} \sin\theta$ ,  $0$ , where M is a constant. Show that the motion is irrotational. Also, find the equation of streamlines. 5+5=10
6. If the lines of motion are curves on the surfaces of cones having the vertices at the origin and the axis of Z for common surface, prove that the equation of continuity is  $\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial r} + \frac{2\rho u}{r} + \frac{\operatorname{cosec} \theta}{r} \frac{\partial (\rho w)}{\partial \varphi} = 0$  where u and w are the velocity components along the increasing direction of r and  $\varphi$ . 10
7. Describe the methods to describe fluid motion. Deduce the differential equation of streamlines. 5+5=10
8. Explain Stretch Ratio. Determine the shear angle  $\gamma_{23}$  for the simple shear displacement 2+8=10

$$x_1 = X_1, x_2 = X_2, x_3 = X_3 + \frac{2X_2}{\sqrt{3}}$$

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