

**M.Sc. MATHEMATICS
FOURTH SEMESTER
GRAPH THEORY
MSM-401**

Duration : 3 hrs.

Full Marks: 70

Time : 20 min.

[PART-A: Objective]

Marks : 20

Choose the correct answer from the following:

1X20=20

- What is the number of edges present in complete graph K_n having n vertices.
 - $\frac{n(n+1)}{2}$
 - $\frac{n(n-1)}{2}$
 - n^2
 - None
- Which of the following is true for any simple connected graph with more than 2 vertices
 - No two vertices have same degree
 - At least two vertices have same degree
 - At least 3 vertices have same degree
 - All vertices have same degree
- A graph is self-complementary if it is isomorphic to its complement. For all self-complementary graphs on n vertices, n is
 - A multiple of 4
 - Even
 - Odd
 - congruent to 0 mod 4, or 1 mod 4
- A graph is said to be isomorphic if
 - The same numbers of vertices
 - The equal numbers of vertices have equal degrees
 - The equal numbers of edges
 - All of the above

5. What is the maximum number of edges in a bipartite graph having 12 vertices?
- a. 24
b. 38
c. 36
d. 32
6. A Graph contains m edges can be decomposed in
- a. $2^m - 1$ ways
b. $2^{m-1} - 1$ ways
c. 2^m ways
d. None
7. The number of Hamiltonian Circuits in a complete graph of n vertices
- a. $\frac{(n-1)!}{2}$
b. $\frac{n!}{2}$
c. $\frac{(n-2)!}{2}$
d. None of these.
8. Conditions of planarity for n numbers of vertices and e numbers of edges
- a. $e < 2n - 6$
b. $e < 2n + 6$
c. $e \leq 3n - 6$
d. None of these
9. Let n_1 and n_2 be the number of edges of graphs G_1 and G_2 respectively, then the number of edges of $G_1 \cup G_2$ if G_1 and G_2 are connected graphs
- a. $n_1 + n_2$
b. $n_1 + n_2 - 1$
c. n_1
d. n_2
10. For a given graph G having v vertices and e edges which is connected and has no cycles, which of the following statements is true?
- a. $v = e$
b. $v = e + 1$
c. $v + 1 = e$
d. None of these

11. Which of the following statements is/are TRUE for undirected graphs?

P: Number of odd degree vertices is even.

Q: Sum of degrees of all vertices is even.

- a. P only
- b. Q only
- c. Both P and Q
- d. Neither P nor Q

12. A cycle on n vertices is isomorphic to its Complement. The value of n is

- a. 3
- b. 4
- c. 5
- d. 6

13. A connected undirected graph containing n vertices and $(n-1)$ edges

- a.
Cannot have cycles
- b. Must contain at least one cycle
- c. Can contain at most two cycles
- d. Must contain at least two cycles

14. A graph is _____ if it has at least one pair of vertices without a path between them.

- a. Completed
- b. Connected
- c. Disconnected
- d. Trivial

15. Total number of trees with 5 vertices

- a. 2
- b. 3
- c. 4
- d. 5

16. What is the radius of the Petersen graph

- a. 2
- b. 3
- c. 4
- d. All of these

17. Let $\kappa(G)$ = vertex connectivity, $\lambda(G)$ = edge connectivity and $\delta(G)$ = minimum degree of graph G. Then

- a. $\kappa(G) \leq \delta(G) \leq \lambda(G)$
- b. $\lambda(G) \leq \delta(G) \leq \kappa(G)$
- c. $\delta(G) \leq \kappa(G) \leq \lambda(G)$
- d. $\kappa(G) \leq \delta(G) < \lambda(G)$

18. Let G be a simple graph with every pair of vertices is connected. Then G is
- a. Trivial
 - b. Complete
 - c. Disconnected
 - d. Self-complementary
19. Let $G = C_n$. Then
- a. There is path of length n
 - b. There is a closed path of length n
 - c. G is bipartite
 - d. None of the above
20. A graph is called a _____ if it is a connected acyclic graph
- a. Cyclic graph
 - b. Regular graph
 - c. Tree
 - d. Trivial graph

(PART-B : Descriptive)

Time: 2 HRS 40 MINS

Marks : 50

[Answer question no.(1) & any four (4) from the rest]

1. a. Write a brief History of Graph Theory. 5+5=10
b. Prove that Simple Graph with n -vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges

2. a. With examples of each define union, intersection, Edge deletion, vertex deletion, fusion of a Graph. 5+5=10
b. Prove that the number of labeled trees with n vertices ($n \geq 2$) is n^{n-2} .

3. a. Show that any connected Graph with n -vertices and $(n-1)$ edges are a tree. 5+5=10
b. Show that the spanning tree in a connected Graph of n vertices and e edges has $(n-1)$ tree branches and $(e-n+1)$ chords

4. 5+5=10
a. Show that The maximum vertex connectivity one can achieve with a graph G of n vertices and e edges, $e \geq n-1$ is the integral part of the number $\frac{2e}{n}$ i.e. $\left\lfloor \frac{2e}{n} \right\rfloor$
b. Show that the ring sum of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-set

5. a. Prove that the complete Graph of five vertices is non-planar. 5+5=10
b. Define complete Graph. Show that in a complete Graph with n vertices there are $\frac{n-1}{2}$ edges disjoint Hamiltonian circuit if n is odd no, $n \geq 3$.

6. a. Prove that -Every tree with two or more vertices is two chromatic. 5+5=10
b. Let a and b two non-adjacent vertices in a Graph G . Let G' be a

graph obtained by adding an edge between a and b . Let G'' be a simple Graph obtained from G by fusing the vertices a and b together and replacing sets of parallel edges with single edges then prove that $P_n(\lambda)$ of $G = P_n(\lambda)$ of G' + $P_{n-1}(\lambda)$ of G''

7. a. If G is a simple Graph with largest vertex Δ then prove that $\Delta \leq x'(G) \leq \Delta + 1$. 5+5=10
- b. Define Trivial Graph, Pseudo graph, Trivial Graph, pendent vertex, isolated vertex. Also define and explain characteristics of isomorphism of graph with a suitable diagram
8. a. Prove that every connected Graph has at most one spanning Tree. 5+5=10
- b. Prove that a connected Graph G is an Euler Graph if all vertices of G are even degree.

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