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Fundamentals of Pharmaceutical Calculations

OBJECTIVES

Upon successful completion of this chapter, the student will be able to:

- Apply the method of *ratio and proportion* in problem solving.
- Apply the method of *dimensional analysis* in problem solving.
- Demonstrate the use of *percent* in pharmaceutical calculations.
- Apply and validate the method of *estimation* in pharmaceutical calculations.

Introduction

Pharmaceutical calculations is the area of study that applies the basic principles of mathematics to the preparation and efficacious use of pharmaceutical preparations. It includes calculations from initial product formulation through clinical administration and outcomes assessment.

Mathematically, pharmacy students beginning use of this textbook are well prepared. The basic units of measurement and problem-solving methods have been previously learned and are familiar. The newness lies in the terminology used and in the understanding of the pharmaceutical/clinical purpose and goal of each computation. *Of vital importance is an appreciation of the need for accuracy, as each calculation must be understood to be directly applicable to the health outcomes and safety of patients.* Therefore, the student must communicate information clearly and accurately. According to the Institute for Safe Medication Practices, a trailing zero should never be used following a decimal point to show accuracy (e.g., 1.0 mL) because it can result in a 10-fold error if the decimal point is not seen (i.e., 10 mL). Similarly, a zero should always precede the decimal point in decimal fractions less than one (e.g., 0.2 mg) to avoid missing the decimal point and also creating a 10-fold error.[1] Rounding of numbers within a calculation should be avoided, and no rounding should be done until the final answer has been calculated to determine the most accurate answer. In most instances rounding the final answer to two or three decimal places is acceptable.

This initial chapter introduces some basic aspects and methods of pharmaceutical calculations.

Units of Measurement

Pharmacy and all other health professions utilize the ***International System of Units (SI)***, commonly referred to as the ***metric system.*** This familiar system, with its base units (*meter, liter, kilogram*) and corresponding subdivisions, is presented in detail in Chapter 2. Pharmaceutical calculations often require the accurate conversion of quantities from a given or calculated unit to another (e.g., *milligrams* to *micrograms*). Proficiency in operating within this system is fundamental to the practice of pharmacy.

Two other systems of measurement are presented in Appendix A. The *avoirdupois system* is the common system of commerce, which has not fully been replaced in the United States by the International System of Units. Many product designations are *dual scale:* that is, equivalent SI and common system measures. It is in the common system that goods are packaged and sold by the ounce, pound, pint, quart, and gallon or linearly measured by the inch, foot, yard, and mile. The *apothecaries' system of measurement* is the traditional system of pharmaceutical measurement, which is now largely of historic significance. *Intersystem conversion* remains an exercise in pharmaceutical calculations and is a component of Appendix A.

Ratio and Proportion

Ratio

The relative amount of two quantities (one to the other), is called their *ratio.* A ratio resembles a common fraction except in the manner in which it is presented. For example, the fraction ½ may be expressed as the ratio 1:2, which is not read as "one half," but rather as "one is to two." Rules governing common fractions apply to ratios. For example, if the two terms of a ratio are either multiplied or divided by the same number, the value remains unchanged. The *value* is the quotient of the first term divided by the second term. For instance, the value of the ratio 20:4 is 5. If the ratio is multiplied by 4, becoming 80:16, or divided by 4, becoming 5:1, the value remains 5. When two ratios have the same value, they are termed ***equivalent ratios***, as is the case with the ratios 20:4, 80:16, and 5:1.

As described next, equivalent ratios provide the basis for problem solving by the *ratio-and-proportion* method.

Proportion

A ***proportion*** is the expression of the equality of two ratios. It may be written in any one of three standard forms:

$$(1)\quad a : b = c : d$$

$$(2)\quad a : b :: c : d$$

$$(3)\quad \frac{a}{b} = \frac{c}{d}$$

Each of these expressions is read: *a is to b as c is to d*, and *a* and *d* are called the *extremes* (meaning "outer members") and *b* and *c* the *means* ("middle members").

In any proportion, *the product of the extremes is equal to the product of the means.* This principle allows us to find the missing term of any proportion when the other three terms are known. If the missing term is a *mean*, it will be *the product of the extremes divided by the given mean*, and if it is an *extreme*, it will be *the product of the means divided by the given extreme.* Using this information, we may derive the following fractional equations:

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then}$$

$$a = \frac{bc}{d},\ b = \frac{ad}{c},\ c = \frac{ad}{b}, \text{ and } d = \frac{bc}{a}.$$

In a proportion that is properly set up, the position of the unknown term does not matter. However, some persons prefer to place the unknown term in the fourth position—that is, in the denominator of the second ratio. *It important to label the units in each position (e.g., mL, mg) to ensure the proper relationship between the ratios of a proportion.*

The application of ratio and proportion enables the solution of many of the pharmaceutical calculation problems in this text and in pharmacy practice.

1. *If 3 tablets contain 975 milligrams of aspirin, how many milligrams should be contained in 12 tablets?*

$$\frac{3 \text{ tablets}}{12 \text{ tablets}} = \frac{975 \text{ milligrams}}{\text{x milligrams}}$$

$$\text{x milligrams} = \frac{12 \text{ tablets} \times 975 \text{ milligrams}}{3 \text{ tablets}} = \mathbf{3900\ milligrams}$$

2. *If 3 tablets contain 975 milligrams of aspirin, how many tablets should contain 3900 milligrams?*

$$\frac{3 \text{ tablets}}{\text{x tablets}} = \frac{975 \text{ milligrams}}{3900 \text{ milligrams}}$$

$$\text{x tablets} = \frac{3 \text{ tablets} \times 3900 \text{ milligrams}}{975 \text{ milligrams}} = \mathbf{12\ tablets}$$

3. *If 12 tablets contain 3900 milligrams of aspirin, how many milligrams should 3 tablets contain?*

$$\frac{12 \text{ tablets}}{3 \text{ tablets}} = \frac{3900 \text{ milligrams}}{\text{x milligrams}}$$

$$\text{x milligrams} = \frac{3 \text{ tablets} \times 3900 \text{ milligrams}}{12 \text{ tablets}} = \mathbf{975\ milligrams}$$

4. *If 12 tablets contain 3900 milligrams of aspirin, how many tablets should contain 975 milligrams?*

$$\frac{12 \text{ tablets}}{\text{x tablets}} = \frac{3900 \text{ milligrams}}{975 \text{ milligrams}}$$

$$\text{x tablets} = \frac{12 \text{ tablets} \times 975 \text{ milligrams}}{3900 \text{ milligrams}} = \mathbf{3\ tablets}$$

Proportions need not contain whole numbers. If common or decimal fractions are supplied in the data, they may be included in the proportion without changing the method. For ease of calculation, it is recommended that common fractions be converted to decimal fractions prior to setting up the proportion.

5. *If one dose of a cough syrup is 1¼ milliliters (mL) for a small child, how many milliliters will be needed for 12 doses of the syrup?*

$$1\tfrac{1}{4} \text{ mL} = 1.25 \text{ mL}$$

$$\frac{1.25 \text{ mL}}{1 \text{ dose}} = \frac{\text{x}}{12 \text{ doses}}$$

$$\text{x} = \mathbf{15\ mL}$$

CALCULATIONS CAPSULE

Ratio and Proportion

- A *ratio* expresses the relative magnitude of two like quantities (e.g., 1:2, expressed as "1 to 2.")
- A proportion expresses the equality of two ratios (e.g., 1:2 = 2:4).
- The four terms of a proportion are stated as:

$$a : b = c : d\text{, or } a : b :: c : d\text{, or } \frac{a}{b} = \frac{c}{d}$$

  and expressed as "*a* is to *b* as *c* is to *d*."
- Given three of the four terms of a proportion, the value of the fourth, or missing, term may be calculated by cross multiplication and solution.
- The ratio-and-proportion method is a useful tool in solving many pharmaceutical calculation problems.

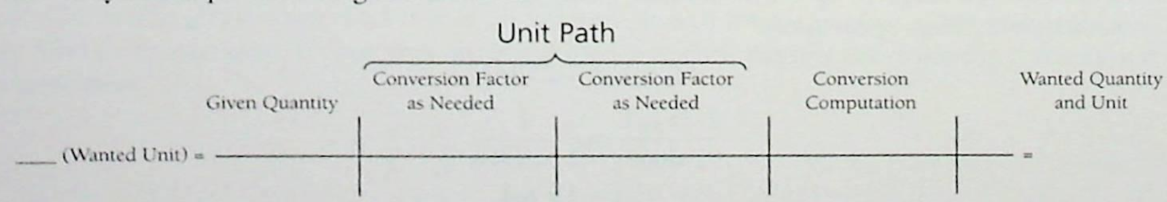
Dimensional Analysis

When performing pharmaceutical calculations, some students prefer to use a method termed ***dimensional analysis*** (also known as ***factor analysis***, ***factor-label method***, or ***unit-factor method***). This method involves the logical sequencing and placement of a series of ratios (termed *factors*) into an equation. The ratios are prepared from the given data as well as from selected *conversion factors* and contain both arithmetic quantities and their units of measurement. Some terms are inverted (to their reciprocals) to permit the cancellation of like units in the numerator(s) and denominator(s) and leave only the desired terms of the answer. One advantage of using dimensional analysis is the consolidation of several arithmetic steps into a single equation.

In solving problems by dimensional analysis, the student unfamiliar with the process should consider the following steps[2,3]:

Step 1. Identify the wanted unit of the answer (e.g., mL, mg, etc.) and place it at the beginning of the equation. Some persons prefer to place a question mark next to it.
Step 2. Identify the given quantity(ies) and its (their) unit(s) of measurement.
Step 3. Identify the conversion factor(s) that is (are) needed for the "unit path" to arrive at the arithmetic answer in the unit wanted.
Step 4. Set up the ratios such that the cancellation of the units of measurement in the numerators and denominators will retain only the wanted unit as identified in *Step 1*.
Step 5. Perform the arithmetic computation by multiplying the numerators, multiplying the denominators, and dividing the product of the numerators by the product of the denominators.

The general scheme shown here and in the "Calculations Capsule: Dimensional Analysis" may be helpful in using the method.



Example calculations using dimensional analysis

1. *How many fluidounces (fl. oz.) are there in 2.5 liters (L)?*
   Step 1. The wanted unit for the answer is *fluidounces*.
   Step 2. The given quantity is 2.5 L.

STEP 3. The conversion factors needed are those that will take us from liters to fluidounces.

As the student will later learn, these conversion factors are as follows:

1 liter = 1000 mL (to convert the given 2.5 L to milliliters)

1 fluidounce = 29.57 mL (to convert milliliters to fluidounces)

STEP 4. Set up the ratios in the unit path.

Unit Path

	Given Quantity	Conversion Factor as Needed	Conversion Factor as Needed	Conversion Computation		Wanted Quantity and Unit
fl.oz. (Wanted Unit) =	2.5 L	1000 mL	1 fl. oz.			=
		1 L	29.57 mL			

NOTE: The unit path is set up such that all units of measurement will cancel out except for the unit wanted in the answer, *fluidounces*, which is placed in the numerator.

STEP 5. Perform the computation:

Unit Path

	Given Quantity	Conversion Factor as Needed	Conversion Factor as Needed	Conversion Computation	Wanted Quantity and Unit
fl.oz. (Wanted Unit) =	$2.5\cancel{L}$	$1000\ \cancel{mL}$	1 fl. oz.	$\frac{2.5 \times 1000 \times 1}{1 \times 29.57} = \frac{2500}{29.57}$	= 84.55 fl. oz.
		$1\ \cancel{L}$	$29.57\ \cancel{mL}$		

or

$$2.5\cancel{L} \times \frac{1000\ \cancel{mL}}{1\ \cancel{L}} \times \frac{1\text{ fl. oz.}}{29.57\ \cancel{mL}} = \frac{2.5 \times 1000 \times 1}{1 \times 29.57} = \frac{2500}{29.57} = \mathbf{84.55\ fl.\ oz.}$$

2. *A medication order calls for 1000 milliliters of a dextrose intravenous infusion to be administered over an 8-hour period. Using an intravenous administration set that delivers 10 drops/milliliter, how many drops per minute should be delivered to the patient?*

$$\frac{1000\ \cancel{mL}}{8\ \cancel{hours}} \times \frac{10\text{ drops}}{1\ \cancel{mL}} \times \frac{1\ \cancel{hour}}{60\text{ minutes}} = 20.83\text{ drops/minute} \approx \mathbf{21\ drops/minute}$$

NOTE: "drops" was placed in the numerator and "minutes" in the denominator to arrive at the answer in the desired term, *drops per minute.*

CALCULATIONS CAPSULE

Dimensional Analysis

- An alternative method to ratio and proportion in solving pharmaceutical calculation problems.
- The method involves the logical sequencing and placement of a series of ratios to consolidate multiple arithmetic steps into a single equation.
- By applying select conversion factors in the equation—some as reciprocals—unwanted units of measure cancel out, leaving the arithmetic result and desired unit.
- Dimensional analysis scheme:

Unit Path

	Given Quantity	Conversion Factor as Needed	Conversion Factor as Needed	Conversion Computation		Wanted Quantity and Unit
___ (Wanted Unit) =						=