REV-01
MSM/28/14/19

2023/01

SET B

M.Sc. MATHEMATICS
FIRST SEMESTER
ABSTRACT ALGEBRA I
MSM – 103

[USE OMR SHEET FOR OBJECTIVE PART]

Duration : 1.30 hrs. **Full Marks : 35**

(Objective)

Time: 15 min. **Marks: 10**

Choose the correct answer from the following: ***1X10=10***

1. Let G be a group of order 56. Then which of the following is/are always false:
   a. All 7-Sylow subgroups of G are normal.
   b. All 7-Sylow subgroups of G are normal.
   c. Either a 7-Sylow subgroup or a 2-Sylow subgroup of G is normal.
   d. There is a proper normal subgroup of G.

2. Let G be a group of order 49. Then
   a. G is Abelian
   b. G is cyclic
   c. G is non-Abelian
   d. Centre of G has order 7.

3. Let G be a group of order 77. Then the center of G is isomorphic to
   a. Trivial group
   b. $\mathbb{Z}_7$
   c. $\mathbb{Z}_{11}$
   d. $\mathbb{Z}_{77}$

4. Let G be a group of order 15. Then the number of Sylow subgroups of G of order 3 is
   a. 0
   b. 1
   c. 3
   d. 5

5. The number of elements in $\text{Aut}(\mathbb{Z}_{25})$ is?
   a. 15
   b. 20
   c. 25
   d. 30

6. Upto isomorphism, the number of Abelian group of order 121 is
   a. 1
   b. 2
   c. 11
   d. 10

7. Which of the following is/are class equation(s)
   a. $14 = 1 + 1 + 2 + 3 + 7$
   b. $8 = 1 + 1 + 3 + 3$.
   c. $22 = 1 + 11 + 2 + 2 + 2 + 2 + 2$
   d. All the above

8. The order of the conjugacy class $\text{cl}\big((12)(45)\big)$ in S_5 is
   a. 15
   b. 10
   c. 25
   d. 30

9. Let G be a group. Suppose G has subgroups of order 45 and 75. If $|G| < 400$, then $|G|$ is equal to

a. 90
b. 150
c. 175
d. 225

10. Let G be a non-Abelian group. Then its order can be:

a. 21
b. 55
c. 25
d. 35

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Descriptive

Time : 1 hrs. 15 mins. Marks : 25

[Answer question no.1 & any two (2) from the rest]

1. a. Prove or disprove that- A_4 has no subgroup of order 6. 3+2=5

b. Find all possible class equation of a group of order 22.

2. a. Determine all homomorphisms from $\mathbb{Z}_{30}$ to $\mathbb{Z}_{12}$. 3+4+3 =10

b. Determine the number of elements of order 5 in $\mathbb{Z}_{25} \times \mathbb{Z}_5$.

c. Find all the cyclic subgroups of $\mathbb{Z}_{10}$.

3. a. Let σ and τ be the permutation defined by 5+3+2 =10

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 3 & 5 & 7 & 9 & 6 & 4 & 8 & 2 \end{pmatrix}$$
$$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 8 & 3 & 4 & 9 & 6 & 5 & 2 & 1 \end{pmatrix}$$

Find the inverse of $\sigma\&\tau$ and find their order.Also, prove or disprove the followings:

(i) σ and τ commute each other.

(ii) $<\sigma>\cap<\tau>$ has order 1.

b. Let $\beta \in S_7$ and suppose $\beta^4 = (2143567)$. Find β.

c. Show that A_5 has 20 elements of order 3, and 15 elements of order 2.

4. a. Let G be a non-Abelian group of order 21. Let H be a Sylow 3-subgroup and K be a Sylow 7-subgroup of G. Prove or disprove that either$H$or$K$is normal in G. 4+3+3 =10

b. Let G be a group of permutation. For each $f \in G$, define

$$\phi(f) = \begin{cases} +1, & \text{if } f \text{ is an even permutation} \\ -1, & \text{if } x \text{ is an odd permutation} \end{cases}$$

Prove that -$\phi: G \to \{+1, -1\}$ is a group homomorphism. Find the kernel.

c. Let G be an Abelian group with identity e. Then prove or disprove that $H = \{x \in G : x^2 = e\}$ is a subgroup of G.

5. a. Suppose that ϕ is a homomorphism from $U(40)$ to $U(40)$ with kernel $\{1, 9, 17, 33\}$. If $\phi(11) = 11$, find all elements of $U(40)$ that map to 11. 2+2+2+2+2=10

b. List all possible Abelian group of order 56 upto isomorphism.

c. Express $U(165)$ as an external direct product of cyclic additive groups of the form $\mathbb{Z}_n$.

d. Prove that - Any group of order 99 is Abelian.

e. Determine the class equation of S_5.

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