REV-01
MSM/49/25/30

2023/06

SET A

M.Sc. MATHEMATICS
FOURTH SEMESTER
ADVANCED PARTIAL DIFFERENTIAL EQUATION
MSM – 402

[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

(Objective)

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1X20=20***

1. Charpit's Auxillary equations for the following non linear partial differential equation $z = px + qy + p^2 + q^2$ is

   a. $\frac{dp}{2p} = \frac{dq}{2q} = \frac{dz}{-p(x-2p)-q(y-2q)} = \frac{dx}{-x+2p} = \frac{dy}{-y+2q}$

   b. $\frac{dp}{0} = \frac{dq}{0} = \frac{dz}{p(x+2p)+q(y+2q)} = \frac{dx}{x+2p} = \frac{dy}{y+2q}$

   c. $\frac{dp}{2p} = \frac{dq}{2q} = \frac{dz}{p(x-2p)+q(y-2q)} = \frac{dx}{-x+2p} = \frac{dy}{-y+2q}$

   d. $\frac{dp}{0} = \frac{dq}{0} = \frac{dz}{p(x-2p)+q(y-2q)} = \frac{dx}{-x+2p} = \frac{dy}{-y+2q}$

2. $\lambda -$ quadratic for the solution of partial differential equation $3r + 4s + t + \left(rt - s^2\right) = 1$ is

   a. $4\lambda^2 - 4\lambda + 1 = 0$

   b. $4\lambda^2 + 4\lambda - 1 = 0$

   c. $4\lambda^2 + 4\lambda + 1 = 0$

   d. $4\lambda^2 - 4\lambda - 1 = 0$

3. In clairaut's form the symbol $p = ?$

   a. $p = \frac{\partial f}{\partial x}$

   b. $p = \frac{\partial y}{\partial x}$

   c. $p = \frac{dy}{dx}$

   d. none of above

4. The degree of the following non linear PDE $\left(\frac{\partial^2 z}{\partial x^2}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^3 = x^2 y^2 z^2$ is

   a. one

   b. Two

   c. Three

   d. Four

5. The complete solution of the equation $q = 3p^2$ is

   a. $z = ax + 3a^2 y + b$

   b. $z = \pm\sqrt{1-b^2}\,x + by + c$

   c. $z = \pm\sqrt{1-b^2}\,x - by + c$

   d. $z = \pm(1-b^2)x - by + c$

6. For Lagrange's form of the first order differential equation $Qp + Rq = P$,the subsidiary equations are

a. $\frac{dx}{Q} = \frac{dy}{P} = \frac{dz}{R}$

b. $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

c. $\frac{dx}{R} = \frac{dy}{P} = \frac{dz}{Q}$

d. $\frac{dx}{Q} = \frac{dy}{R} = \frac{dz}{P}$

7. The equation $xp + yq = z$ is

a. non linear

b. clairaut's equation

c. linear

d. none of above

8. The General partial differential equation of second order for a function of two independent variable

$Rr + Ss + Tt + f(x, y, z, p, q) = 0$ is hyperbolic if

a. $S^2 - 4RT < 0$

b. $S^2 - 4RT = 0$

c. $S^2 - 4RT \neq 0$

d. $S^2 - 4RT > 0$

9. For two dimensional wave equation,the finding deflection is

a. $u(x, y, t)$ at (x, y) at any time $t > 0$

b. $u(x, t)$ at (x, y) at any time $t > 0$

c. $u(x, y, t)$ at x-axis at any time $t > 0$

d. None of the above

10. The partial differential equation

$u_{xx} + u_{yy} = u_{zz}$ is

a. Parabolic type

b. Elliptic type

c. Hyperbolic type

d. none of above

11. $f(p, q) = 0$ is known as

a. Standard form II

b. Standard form I

c. Standard form III

d. Standard form IV

12. The Characteristics of the equation $y^2 r - x^2 t = 0$ are

a. $x^2 + y^2 = c_1, x^2 - y^2 = c_2,$

b. $x + y = c_1, x - y = c_2,$

c. $x^4 + y^4 = c_1, x^2 - y^2 = c_2,$

d. $x^2 + y^2 = c_1, x^4 - y^4 = c_2,$

13. Monge's subsidiary equations for
$(r-t)xy - s(x^2 - y^2) = qx - py$ are

a. $xdpdy - ydqdx + (py - qx)dxdy = 0$
$x(dy)^2 - (x^2 - y^2)dxdy - y(dx)^2 = 0$

b. $ydpdy - xdqdx - (px - qy)dxdy = 0$
$y(dy)^2 - xdxdy + xy(dx)^2 = 0$

c. $xydpdy - xydqdx + (py - qx)dxdy = 0$
$xy(dy)^2 - (-x^2 + y^2)dxdy - xy(dx)^2 = 0$

d. None of the above

14. The $\lambda -$ quadratic equation for a general partial differential equation
$Rr + Ss + Tt + f(x, y, z, p, q) = 0$ is

a. $R\lambda^2 + S\lambda + T > 0$
b. $R\lambda^2 - S\lambda - T = 0$
c. $R\lambda^2 + S\lambda + T < 0$
d. $R\lambda^2 + S\lambda + T = 0$

15. The one-dimensional wave equation is

a. $\frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial t^2}, c^2 = \frac{T}{\rho}$
b. $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, c^2 = \frac{T}{\rho}$
c. $\frac{\partial u}{\partial t} = c^2 \frac{\partial u}{\partial x}, c^2 = \frac{T}{\rho}$
d. $\frac{\partial u}{\partial x} = c^2 \frac{\partial u}{\partial t}, c^2 = \frac{T}{\rho}$

16. Solutions of a partial differential equation $r = 6x$ is

a. $z = x^3 - x\phi_1(y) + \phi_2(y)$
b. $z = x^3 - x\phi_1(y) - \phi_2(y)$
c. $z = x^3 + x\phi_1(y) + \phi_2(y)$
d. $z = -x^3 + x\phi_1(y) + \phi_2(y)$

17. For the partial differential equation
$3r + 4s + t + (rt - s^2) = 1$, there exist

a. No any intermediate integral
b. Only one intermediate integral
c. Possibly two intermediate integrals
d. Exactly two intermediate integrals

18. The equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, k = \frac{K}{\rho\sigma}$ is called

a. One dimensional Heat equation
b. two dimensional Heat equation
c. One dimensional fourier equation
d. two dimensional fourier equation

19. Solution of the equation $\frac{\partial^2 v}{\partial x^2} = \frac{\partial v}{\partial t}$, $v = 0$ when $t \to \alpha$ as well as $v = 0$ at $x = 0$ and $x = l$ is

a. $v(x,t) = \sum_{n=1}^{\alpha} B_n Sin \frac{n\pi x}{l} e^{\frac{-n^2\pi^2 t}{l^2}}$

b. $v(x,t) = \sum_{n=1}^{\alpha} B_n Sin \frac{n\pi x}{l} e^{\frac{n^2\pi^2 t}{l^2}}$

c. $v(x,t) = \sum_{n=1}^{\alpha} B_n Sin \frac{n\pi x}{l} e^{\frac{-n^2\pi t}{l^2}}$

d. $v(x,t) = -\sum_{n=1}^{\alpha} B_n Sin \frac{n\pi x}{l} e^{\frac{-n^2\pi^2 t}{l^2}}$

20. Two dimensional heat flow equation in steady state reduces to

a. $\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

b. $\frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial z^2} = 0$

c. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

d. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

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Descriptive

Time : 2 hrs. 30 mins. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. Classify the partial differential equation $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} = 0$. 2+8=10

   Also Reduce the differential equation $t - s + p - q\left(1 + \frac{1}{x}\right) + \frac{z}{x} = 0$ to canonical form

2. Obtain the two intermediate integrals for the second order partial differential equation 3+3+2 =8

   $t - r\sec^4 y = 2q\tan y$

   Hence obtain the general solution of the equation.

3. Write down the $\lambda -$quadratic for the partial differential equation $Rr + Ss + Tt + U\left(rt - s^2\right) = V$ .Also mention the subsidiary conditions for solution of the equation. Hence solve the partial differential equation 2+2+6 =10

   $5r + 6s + 3t + 2\left(rt - s^2\right) + 3 = 0$

4. Solve 5+5=10

   (a) $x^2p^2 + y^2q^2 = z^2$

   (b) $p + q = pq$

5. Solve the Boundary value problem 10

   $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ if $u(0, y) = 8e^{-3y}$

6. What is the definition of Laplace equation in three dimension. 1+9=10

   Prove that

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

7. Solve 2+2+6 =10

(a) $x^2p^2 + y^2q^2 = z^2$

(b) $p + q = pq$

(c)Find a complete,singualar and general integral of

$\left(p^2 + q^2\right)y = qz$

8. Prove that Charpit's auxillary equations are 6+4=10

$$\frac{dp}{f_x + pf_z} = \frac{dq}{f_y + qf_z} = \frac{dz}{-pf_p - qf_q} = \frac{dx}{-f_p} = \frac{dy}{-f_q}.$$

Solve by Charpit's Method

$2(z + px + qy) = yp^2$

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