

M.Sc. MATHEMATICS
FOURTH SEMESTER
GENERAL MATHEMATICS-II
MSM-406

Duration : 3 hrs.

Full Marks: 70

Time : 20 min.

[PART-A: Objective]

Marks : 20

Choose the correct answer from the following:

1X20=20

1. Which of the following is not a group?

a. (\mathbb{R}, \cdot)

b. $(\mathbb{Z}, +)$

c. $(\mathbb{R}, +)$

d. None of these

2. Which of the following is true?

a. Identity element of a group is always unique.

b. Inverse element of a group is always unique.

c. Both (a) and (b)

d. None of these

3. Which of the following is an Abelian Group

a. The group of all 2×2 invertible matrix w.r.t matrix multiplication.

b. The group $\{1, -1, i, -i\}$ w.r.t multiplication.

c. Both (a) and (b).

d. None of these

4. Let G be a group and H be a subgroup of G . If order of H is 6, then order of G can be

a. 2

b. 12

c. 15

d. None of these.

5. Let $G = \{0, 1, 2\}$. For $a, b \in G$, a binary operation $*$ define on G as $a * b = |a - b|$.

The $(G, *)$ is

a. a group

b. an Abelian group

c. not a group

d. none of these

6. The set of natural number \mathbb{N} is

a. bounded

b. bounded below but not bounded above

c. bounded above but not bounded below

d. not bounded

7. The infimum and supremum of the set $\{-1, -\frac{1}{2}, -\frac{1}{3}, \dots\}$ are respectively

a. -1 and 0

b. 0 and -1

c. Does not exist

d. None of these

8. Consider the following statements:

P: The set \mathbb{R} of real numbers is the neighbourhood of each of its points.

Q: The set \mathbb{Q} of rational numbers is not the neighbourhood of each of its points.

- a. P true, Q false
 c. Both P and Q are true.
- b. P false, Q true
 d. Both P and Q are false.
9. Which of the following is/are not open set?
- a. The set of real numbers, \mathbb{R}
 c. The null set \emptyset
- b. The set of natural numbers, \mathbb{N}
 d. None of these
10. Consider the following statements:
 P: Every finite set of numbers is bounded.
 Q: Any interval in the real line \mathbb{R} is bounded.
- a. P true, Q false
 c. Both P and Q are true.
- b. P false, Q true
 d. Both P and Q are false.
11. Let G be the set of all 2×2 square matrices. The identity element of $(G, +)$ is
- a. 0
 c. Zero matrix
- b. 1
 d. Unit matrix
12. Which of the following is/are open sets?
- a. $[1, \alpha)$
 c. $(0, 1)$
- b. $(-\alpha, 0]$
 d. $[0, 1]$
13. If $f(x) = x^2 \sin \frac{1}{x}$, $x \neq 0$, then the value of the function $f(x)$ at $x = 0$, so that the function is continuous at $x = 0$ is
- a. 0
 c. -1
- b. -1
 d. None of these
14. The set of points where the function f given by $f(x) = |2x - 1| \sin x$ is differentiable is
- a. \mathbb{R}
 c. $(0, \infty)$
- b. $\mathbb{R} - \left\{ \frac{1}{2} \right\}$
 d. None of these
15. The function $f(x) = \cot x$ is discontinuous on the set
- a. $\{x = n\pi, n \in \mathbb{Z}\}$
 c. $\left\{ x = (2n + 1) \frac{\pi}{2}, n \in \mathbb{Z} \right\}$
- b. $\{x = 2n\pi, n \in \mathbb{Z}\}$
 d. None of these
16. The function $f(x) = \frac{4-x^2}{4x-x^2}$ is
- a. Discontinuous at only one point at $x = 0$.
 c. Discontinuous at exactly three points
- b. Discontinuous at exactly two points
 d. None of these
17. If $f(x) = \sqrt{25 - x^2}$, then $\lim_{x \rightarrow 2} \frac{f(x) - f(1)}{x - 1}$ is equal to
- a. $\frac{1}{24}$
 c. $-\sqrt{24}$
- b. $\frac{1}{5}$
 d. $\frac{1}{\sqrt{24}}$

18.

Consider function $f(x) = \begin{cases} x, 0 \leq x \leq 1 \\ 2-x, 1 < x \leq 2 \end{cases}$

- a. f is continuous at $x = 1$ b. f is not continuous at $x = 1$
c. f is continuous at $x = 0$ d. f is not continuous at $x = 0$

19. Consider the following statement:

P: Every continuous function is differentiable.

Q: Every differentiable function is continuous.

- a. P true, Q false b. P false, Q true
c. Both P and Q are true d. Both P and Q are false

20. Which of the following function is not differentiable?

- a. $f(x) = |x|, \forall x \in \mathbb{R}$ b. $f(x) = \sin x, \forall x \in \mathbb{R}$
c. $f(x) = x^3 + 3, \forall x \in \mathbb{R}$ d. None of these

(PART-B : Descriptive)

Time: 2 HRS 40 MINS

Marks : 50

[Answer question no.(1) & any four (4) from the rest]

1. a. Show that the set $\{\pm 1, \pm i, \pm j, \pm k\}$, where $i^2 = j^2 = k^2 = -1$ and $ij = -ji = k, jk = -kj = i, ki = -ik = j$ is a group w.r.t multiplication. Is this an Abelian group? 4+6=10
- b. Find the derivative if
- (i) $y = \sec^{-1} \frac{\sqrt{x}+1}{\sqrt{x}-1} + \sin^{-1} \frac{\sqrt{x}-1}{\sqrt{x}+1}$ $\leq 0 \Rightarrow \frac{2}{(1+t^2)}$
- (ii) $\sin x = \frac{2t}{1+t^2}, \cos y = \frac{1-t^2}{1+t^2}$ $du/dv = 1$
2. Find infimum and supremum of the following sets. Which of the them are bounded? 2x5=10
- (i) $\left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$
- (ii) $\left\{ -2, -\frac{3}{2}, -\frac{4}{2}, -\frac{5}{4}, \dots, -\frac{n+1}{n}, \dots \right\}$
- (iii) $\{x \in \mathbb{R} : 1 < x < 2\}$
- (iv) $\{1, 3, 5, 7, \dots\}$
- (v) $\{1 + (-1)^n : n \in \mathbb{N}\}$
3. Find the derived set of the following sets: 3+3+2=10
- (i) $\{x : 0 \leq x \leq 1\}$
- (ii) $\left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$
- (iii) \mathbb{Z} , the set of integers
- (iv) \mathbb{Q} , the set of rational numbers
4. Let G be an Abelian group. Prove or disprove that the following sets are subgroup of G : 5+5=10
- (i) $H = \{x^2 : x \in G\}$
- (ii) $H = \{x \in G : x^2 = e\}$, where e is the identity of G .

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5. Solve

5+5=10

a. $\int \frac{dx}{x^3 - x^2 - x + 1}$

b. $\int \frac{(x-1)e^x}{(x+1)^2} dx$

6. Find $\frac{dy}{dx}$

2x5=10

(a) $\sqrt{\frac{(x-2)(x-3)}{(x-4)(x-5)(x-6)}}$

(b) $\sin x^{\cos x} + \cos \sqrt{1-x^2}$

7. Examine the continuity of the function

10

$$f(x) = \begin{cases} 3, & 0 \leq x \leq 1 \\ 4, & 1 < x < 3 \\ 5, & 3 \leq x \leq 10 \end{cases}$$

8. Find the values of a and b if the function $f(x)$ defined by

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$$f(x) = \begin{cases} x^2 + 3x + a, & x \leq 1 \\ bx + 2, & x > 1 \end{cases}$$

is differentiable at $x = 1$.

$a=3, b=5.$

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5 (6)