M. Sc. Physics
FIRST SEMESTER
MATHEMATICAL PHYSICS-I
MPH-101

Duration: 3 Hrs. **Marks: 70**

{ **Part : A (Objective) = 20**
Part : B (Descriptive) = 50 }

[PART-B : Descriptive]

Duration: 2 Hrs. 40 Mins. **Marks: 50**

[Answer question no. One (1) & any four (4) from the rest]

1. a) Write the characteristic equation of the matrix A. 1+4+5 =10

b) Find the Eigen values of the matrix $\begin{bmatrix} 4 & -2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$.

c) Find the characteristic equation of the matrix $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$ and the find the value of A^{-1}.

2. a) Define hermitian and skew hermitian matrices. 2+1+2+5 =10

b) What is the necessary and sufficient condition for a matrix to be hermitian.

c) Show that $A = \begin{bmatrix} 1 & 1-i & 2 \\ 1+i & 3 & i \\ 2 & -i & 0 \end{bmatrix}$ is hermitian.

d) Express the matrix $A = \begin{bmatrix} 1+i & 2 & 5-5i \\ 2i & 2+i & 4+2i \\ -1+i & -4 & 7 \end{bmatrix}$ as the sum of hermitian and skew hermitian matrices.

3. a) Find the Laplace transformation of $(1 + sin2t)$. 5+5=10

b) Using Laplace transformation find the initial value problem
$y'' - 4y' + 4y = 64sin2t;\ y(0) = 0,\ y'(0) = 1$.

4. a) Show that any tensor of rank 2 can be expressed as a sum of a symmetric and an antisymmetric tensor, both of rank 2. 4+2+4 =10

b) If A^{μ} and are any two vectors, one contravariant and the other covariant, prove that their outer product is invariant.

c) Using the tensor identity $\in_{ijk}\in_{ipq} = \delta_{jp}\delta_{kq} - \delta_{jq}\delta_{kp}$, prove the following vector identity

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B}).$$

5. a) Write down the components of metric tensor in spherical polar coordinate. Show that $dg_{\alpha\beta} = -g_{\mu\alpha} g_{\nu\beta} dg^{\mu\nu}$. 2+4+2+2 =10

b) Define Christofell's symbols of first and second kind. Show that

$$\Gamma^{\sigma}_{\mu\nu} = g^{\sigma\lambda}\Gamma_{\lambda,\mu\nu}.$$

6. Obtain a solution of Laplace's equation in spherical polar coordinates. 10

7. (a) For z=x+iy, verify if the function (1/z) is analytic or not? 3+7=10

(b) Show that for a function f(z); where z=x+iy,

$$\oint_c f(z)dz = \oint_{c1} f(z)dz\,;$$

Where c and c_1 are two closed concentric contours of radius R and r, (R>r).

8. If f(z) is an analytical complex function encompassing the area inside the contour *C*, then proof that 10

$$f(z) = 1/2\,\pi i[\oint \frac{f(z)}{z-a} dz],$$

for the circle being traverse counterclockwise.

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M. Sc. Physics
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MATHEMATICAL PHYSICS - I
MPH - 101

[PART-A : Objective]

Choose the correct answer from the following: 1×20=20

1. A square matrix A is said to be orthogonal if it follows
   a. $A + A^T = I$
   b. $A.A^T = I$
   c. $\frac{A}{A^T} = I$
   d. $(A + \bar{A})^T = I$

2. If A and B are two square matrices of same order, such that $AB = BA = I$, then the vectors are _________ of each other. (Fill in the black).
   a. transpose
   b. inverse
   c. orthogonal
   d. conjugate

3. The Eigen values of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ are
   a. 1, 0
   b. 1, 1
   c. 1, 2
   d. 0, 2

4. The necessary and sufficient condition for matrix A to be Hermitian is
   a. $A = \bar{A}$
   b. $A = \bar{A}^T$
   c. $A = A^T$
   d. $A = A^{-1}$

5. If A and B are two square matrices of same order, and if there exist a non-singular matrix P, then similarity transformation holds the following relation
   a. $B = AP$
   b. $B = AP^{-1}$
   c. $B = P^{-1}AP$
   d. $B = (PA)^{-1}P$

6. The two vectors X and Y in Real space are orthogonal if they follow the relation
   a. $X.Y = 0$
   b. $X.Y = 1$
   c. $X.Y = \frac{1}{2}$
   d. $X.Y = \frac{\pi}{2}$

7. Laplace transformation of e^{at} is
   a. $\frac{1}{a}$
   b. $\frac{1}{a+t}$
   c. $\frac{1}{s-a}$
   d. $\frac{1}{s+a}$

8. Inverse Laplace transformation of $\frac{1}{s}$ is
   a. 0
   b. 1
   c. ∞
   d. e^{-st}

9. The number of independent components of an antisymmetric tensor of rank 2 in n-dimensional space is
   a. n^2
   b. $\frac{n(n+1)}{2}$
   c. $n+1$
   d. $\frac{n(n-1)}{2}$

10. The contraction of a tensor A^p_{mn} produces a
   a. a scalar
   b. a covariant tensor of rank 2
   c. a vector
   d. a mixed tensor of rank 2

11. The value of the identity $\delta_{ik}\varepsilon_{ikm}$ is
   a. 3
   b. 0
   c. -1
   d. +1

12. The condition of orthogonality of two tensors A^μ and B_ν of rank 1 is given by
   a. $g_{\mu\nu}A^\mu B^\nu = 0$
   b. $g_{\mu\nu}A^\mu B^\nu = 1$
   c. $g_{\mu\nu}A^\mu B^\nu = -1$
   d. none of the above

13. If $g_{\mu\nu} = 0$ for $\mu \neq \nu$ and μ, ν, σ are unequal indices, then the value of $\Gamma^\nu_{\mu\mu}$ is
   a. 0
   b. $-\frac{1}{2g_{\nu\nu}}\frac{\partial g_{\mu\mu}}{\partial x^\nu}$
   c. $\frac{1}{2}\frac{\partial g_{\mu\mu}}{\partial x^\nu}$
   d. $\frac{1}{2}\frac{\partial g_{\mu\nu}}{\partial x^\mu}$

14. Which of the following equations represents the Laplace's equation is
   a. $\Box^2\phi = 0$
   b. $\nabla^2\phi = \rho$
   c. $\nabla^2\phi = 0$
   d. $\nabla^2\phi = k^2\phi$

15. If a function $f(x)$ is defined at $x = 0$, then the value of the expression $\int_{-\infty}^{+\infty} f(x)\,\delta(x)\,dx$ is (here, $\delta(x)$ is the Dirac-Delta function)

a. 1
b. $f(0)$
c. $-f'(0)$
d. 0

16. Which of the following statement is not true for the Green's function $G(x,t)$
a. $G(x,t)$ is a continuous function of x
b. The first derivative of Green's function is a discontinuous function
c. Green's function is discontinuous at $x = t$
d. Green's function is a characteristics of the given boundary conditions

17. If v(x, y)+iu(x, y) a complex function for a complex variable z=x+iy, then the Cauchy Riemann conditions are
a. $\frac{\partial u}{\partial v} = \frac{\partial v}{\partial u}$
b. $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$
c. $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$
d. $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

18. According to Cauchy's integral theorem
a. $f(z) = 1/2\,\pi i[\oint \frac{f(z)}{z-a} dz]$
b. $\oint f(z)dz = 0$
c. $\oint f(z)dz = 1,$
d. $\oint f(z)dz = \alpha$

19. Example of a complex function is
a. z=x+iy,
b. z=x+y,
c. U(x,y)+iV(x, y),
d. U(x,y)+V(x, y),

20. Example of a complex variable
a. z=x+iy,
b. z=x+y,
c. U(x,y)+iV(x, y),
d. U(x,y)+V(x, y),

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UNIVERSITY OF SCIENCE & TECHNOLOGY, MEGHALAYA

[PART (A) : OBJECTIVE]

Duration : 20 Minutes

Serial no. of the main Answer sheet

Course : ....

Semester : .... **Roll No :** ....

Enrollment No : .... **Course code :** ....

Course Title : ....

Session : 2017-18 **Date :** ....

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Instructions / Guidelines

- The paper contains twenty (20) / ten (10) questions.
- Students shall tick (✓) the correct answer.
- No marks shall be given for overwrite / erasing.
- Students have to submit the Objective Part (Part-A) to the invigilator just after completion of the allotted time from the starting of examination.

Full Marks	Marks Obtained
20	

Scrutinizer's Signature *Examiner's Signature* *Invigilator's Signature*