REV-01
MSM/25/30

2024/05

SET A

M.SC. MATHEMATICS
SECOND SEMESTER
ABSTRACT ALGEBRA II
MSM – 203

[USE OMR FOR OBJECTIVE PART]

Duration: 1:30 hrs. **Full Marks: 35**

(Objective)

Time: 15 mins. **Marks: 10**

Choose the correct answer from the following: ***1×10=10***

1. Which of the following statements is/are not necessarily true?
   a. A group of order 4 is solvable
   b. A group of order 15 is solvable
   c. A group of order 25 is solvable
   d. None of these

2. Which of the following groups has no composition series?
   a. Any group order 15
   b. Any group order 64
   c. The ring of integers.
   d. None of these

3. The number of maximal ideals in $\mathbb{Z}_{27}$ is
   a. 0
   b. 1
   c. 2
   d. 3

4. Let p, q be distinct primes. Then $\frac{\mathbb{Z}}{p^2q\mathbb{Z}}$ has
   a. exactly 3 distinct ideals.
   b. exactly 3 distinct prime ideals.
   c. exactly 2 distinct prime ideals.
   d. a unique maximal ideal.

5. The number of homomorphisms from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{28}$ is
   a. 1
   b. 5
   c. 2
   d. 7

6. Let $f(x) = x^3 + 2x^2 + x - 1$. Determine in which of the following cases f is irreducible over the field K where
   a. $K = \mathbb{R}$, the field of real numbers
   b. $K = \mathbb{Q}$, the field of rational numbers
   c. $K = \mathbb{Z}_3$, the field of rational numbers
   d. All the above

7. $\frac{\mathbb{Z}_2[x]}{\langle x^3+x^2+1 \rangle}$ is
   a. a field having 8 elements
   b. a field having 9 elements
   c. an infinite field
   d. Not a field

8. Consider the following statements:
   P: Every principal ideal domain is a Euclidean domain.
   Q: Every Euclidean domain is a unique factorization domain.
   Choose the correct option
   a. P true Q false
   b. P false Q true
   c. Both P and Q are true
   d. Both P and Q are false

9. Consider the following two statements:
P: For any , the group $\mathbb{Z}$ is nilpotent.
Q: A group of order 125 is nilpotent.
Choose the correct option
a. P true Q false
b. P false Q true
c. Both P and Q are true
d. Both P and Q are false

10. Which of the following is/are true?
a. , ≥ 5 is solvable but not nilpotent.
b. , ≥ 5 is nilpotent but not solvable.
c. , ≥ 5 is both solvable and nilpotent.
d. , ≥ 5 is neither solvable nor nilpotent.

-- --- --

(Descriptive)

Time : 1 hr. 15 mins. **Marks : 25**

[Answer question no.1 & any two (2) from the rest]

1. Prove that $1+\sqrt{-3}$ is an irreducible but not prime in $\mathbb{Z}[\sqrt{-3}]$. **5**

2. a. Find all the composition series of a cyclic group $G =< g >$ of order 12 and show that they are equivalent. **4+3+3 =10**

   b. Show that - Any p-group is Nilpotent group. Is a p-group solvable? Justify your answer.

   c. Show that - A group of order pq, where p, q are primes is solvable.

3. a. Determine all the ring homomorphisms from $\mathbb{Z}_{30}$ to $\mathbb{Z}_{20}$. **4+3+3 =10**

   b. Using Fundamental Theorem of ring homomorphism prove that $\frac{\mathbb{Z}_n[x]}{<x>}$ is isomorphic to $\mathbb{Z}_n$, for any integer

   c. Determine all the elements of $\frac{\mathbb{Z}[i]}{<2-i>}$. What is the characteristic of $\frac{\mathbb{Z}[i]}{<2-i>}$?

4. a. Show that - A group of order pq is solvable, where p, q are primes. **4+3+3 =10**

   b. Prove that $I =< 2+2i >$is not a prime ideal of $\mathbb{Z}[i]$. How many elements are in $\frac{\mathbb{Z}[i]}{I}$? What is the characteristic of $\frac{\mathbb{Z}[i]}{I}$?

   c. Let $\mathbb{R}[x]$ denote the ring of all polynomials with real coefficients. Prove that the mapping $\phi: \mathbb{R}[x] \to \mathbb{R}$ defined as $f(x) \mapsto f(1)$ is a ring homomorphism. Also, find the kernel of the homomorphism.