REV-01
MSP/17/22

2024/05

SET A

M.Sc. PHYSICS
FOURTH SEMESTER
THEORY OF RELATIVITY
MSP - 402C
[USE OMR FOR OBJECTIVE PART]

Duration: 3 hrs. **Full Marks: 70**

(Objective)

Time: 30 min. **Marks: 20**

Choose the correct answer from the following: ***1X20=20***

1. The number of independent components of curvature tensor will be
   a. 12
   b. 6
   c. 20
   d. 256

2. The vacuum field equations are defined by
   a. $G_{\mu\nu} = T_{\mu\nu}$
   b. $R_{\mu\nu} = T_{\mu\nu}$
   c. $G_{\mu\nu} = 0$
   d. $R_{\mu\nu} = 0, R \neq 0$

3. The energy-momentum tensor for pressureless perfect fluid will be
   a. $T_{\mu\nu} = \rho\, k_\mu\, k_\nu$
   b. $T_{\mu\nu} = \rho\, U_\mu\, U_\nu$
   c. $T_{\mu\nu} = (\rho + p)k_\mu\, k_\nu + p\, g_{\mu\nu}$
   d. $T_{\mu\nu} = (\rho + p)U_\mu\, U_\nu + p\, g_{\mu\nu}$

4. The Hubble parameter $H(t)$ is defined by
   a. $\dot{a}(t)/a(t)$
   b. $a(t)/\dot{a}(t)$
   c. $\dot{a}(t)$
   d. $a(t)$

5. The metric component g_{rr} in Schwarzschild vacuum solution will be
   a. $\left(1 - \frac{2M}{r}\right)$
   b. r^2
   c. $r^2 \sin^2\theta$
   d. $\left(1 - \frac{2M}{r}\right)^{-1}$

6. The Christoffel symbol $\Gamma^{\theta}_{r\theta}$ is Schwarzschild vacuum solution will be
   a. $1/r^2$
   b. $-1/r^2$
   c. $1/r$
   d. $-1/r$

7. The metric component g_{rr} in FRW-model for a negative curvature will be
   a. $a^2(t)/(1 + r^2)$
   b. $a^2(t)/r$
   c. $a^2(t)/(1 - r)$
   d. $\frac{a^2(t)}{(r-1)}$

8. In Schwarzschild vacuum solution, the conserved quantity angular momentum L in the equatorial plane is defined by
   a. $r\frac{d\phi}{d\tau}$
   b. $r^2\frac{d\phi}{d\tau}$
   c. $\frac{1}{r}\frac{d\phi}{d\tau}$
   d. $\frac{1}{r^2}\frac{d\phi}{d\tau}$

9. In case of zero pressure in FRW-model, the scalar factor and energy-density is related by

a. $a^2\rho = const$ b. $a^2/\rho = const$

c. $a^3\rho = const$ d. $a^3/\rho = const$

10. In Schwarzschild vacuum solution the product $g_{tt}\, g_{rr}$ will be

a. $\left(1-\frac{2M}{r}\right)^2$ b. -1

c. 1 d. $\left(1-\frac{2M}{r}\right)^{-2}$

11. The correct relation of energy-density and pressure in FRW-model will be

a. $\kappa\rho = -6\, a^{''}/a$ b. $\kappa(\rho+3p) = -6\, a^{'}/a$

c. $\kappa(\rho-3p) = -6\, a^{''}/a$ d. $\kappa(\rho+3p) = -6\, a^{''}/a$

12. The equation of orbit for a test particle in the presence of gravitational field produced by Schwarzschild vacuum solution will be

a. $\frac{d^2u}{d\phi^2} = 3GMu^2 + GM/L^2$ b. $\frac{d^2u}{d\phi^2} + u = 3GMu^2 + GM/L^2$

c. $\frac{d^2u}{d\phi^2} + u = GM/L^2$ d. $\frac{d^2u}{d\phi^2} + u = 3GMu^2$

13. In FRW-model, the scalar factor $a(t)$ for $p = 0$ and zero curvature case is

a. $a(t) \propto t^{-2/3}$ b. $a(t) \propto t^{-1/3}$

c. $a(t) \propto t^{2/3}$ d. $a(t) \propto t^{4/3}$

14. In AdS_4 space, the Ricci tensor is given by

a. $-\frac{3}{l^2}\, g_{\mu\nu}$ b. $\frac{3}{l^2}\, g_{\mu\nu}$

c. $-\frac{l^2}{3}\, g_{\mu\nu}$ d. $\frac{l^2}{3}\, g_{\mu\nu}$

15. In Vaidya radiating solution, the energy density is given by

a. $\frac{M^{'}(u)}{\kappa\, r}$ b. $\frac{M^{'}(u)}{\kappa\, r^2}$

c. $-\frac{M^{'}(u)}{\kappa}$ d. $-\frac{M^{'}(u)}{\kappa\, r^2}$

16. In tetrad formalism of four vectors $(k, l, m, \overline{m})$, the correct relation is

a. $k^\mu l_\mu = 1$ b. $k^\mu l_\mu = -1$

c. $k^\mu l_\mu = 0$ d. $k^\mu l_\mu = 1/2$

17. The metric tensor $g^{\mu\nu}$ in terms of the null tetrads in signature (-,+,+,+) is given by

a. $k^\mu l^\nu + l^\mu k^\nu + m^\mu\, \overline{m}^\nu + \overline{m}^\mu\, m^\nu$ b. $-k^\mu l^\nu - l^\mu k^\nu - m^\mu\, \overline{m}^\nu - \overline{m}^\mu\, m^\nu$

c. $-k^\mu l^\nu - l^\mu k^\nu + m^\mu\, \overline{m}^\nu + \overline{m}^\mu\, m^\nu$ d. $-k^\mu m^\nu - l^\mu k^\nu + k^\mu\, \overline{m}^\nu + \bar{l}^\mu\, m^\nu$

18. In Newmann-Penrose formalism, the nonzero Weyl scalar for Petrov type-N metric will be

a. $\Psi_2 \neq 0$ b. $\Psi_3 \neq 0$

c. $\Psi_0 \neq 0$ d. $\Psi_4 \neq 0$

19. In the equatorial plane, the Kretschamnn scalar for Kerr metric is

a. $48\,M^2/r^6$
b. $48\,M^2/r^4$
c. $48\,M^2/r^2$
d. $48\,M/r^6$

20. The horizon radius of Kerr metric in the equatorial plane will be

a. $-M + \sqrt{M^2 - a^2}$
b. $-M - \sqrt{M^2 - a^2}$
c. $M \pm \sqrt{M^2 - a^2}$
d. $2M \pm \sqrt{M^2 - a^2}$

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(Descriptive)

Time : 2 hrs. 30 min. Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. Derive Einstein field equations with cosmological constant using Einstein-Hilbert action. 10

2. Discuss the geodesics motion of test particles around a Schwarzschild vacuum solution. 10

3. What do you mean by FRW model? Derive the basic equations of this FRW model in terms of Hubble parameter. 2+8=10

4. What do you mean by null radiation? Derive a non-static Vaidya radiating solution and discuss the result. 2+8=10

5. What do you mean by tetrad formalism? Construct a null tetrad for the Schwarzschild solution and verify the results. 4+6=10

6. Discuss the physical interpretations and Weyl scalars of space-time geometry. 10

7. Derive the gravitational red shift phenomena of a rotating Kerr metric. Analyze the result for a static body as a particular case. 8+2=10

8. State the shortcomings of general theory of relativity. Discuss various energy conditions associated with the energy-momentum tensor $T_{\mu\nu}$ 4+6=10

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