

M.SC. MATHEMATICS
FIRST SEMESTER
LINEAR ALGEBRA
MSM – 102 [SPECIAL REPEAT]
[USE OMR FOR OBJECTIVE PART]

**SET
A**

Duration: 3 hrs.

Full Marks: 70

Time: 30 min.

[Objective]

Marks: 20

Choose the correct answer from the following:

1×20=20

- Let V be a vector space of all real numbers R over R . Which of the following is vector subspace of V ?
 - $C(R)$
 - $R(R)$
 - $Q(R)$
 - $Z(R)$
- Which one of the following polynomials lies in the linear span of $S = \{1, 1 + x + x^2\}$?
 - $5x^2 + 5x + 1$
 - $100x^2 + 10x + 10$
 - $\pi(x^2 + 1)$
 - $7x^2 + \pi x + 3$
- The singleton set $\{\alpha\}$ is linearly dependent iff
 - $\alpha = 0$
 - $\alpha \neq 0$
 - α is scalar
 - None
- If W is the proper subspace of a finite dimensional vector space V , then
 - $\dim V < \dim W$
 - $\dim W < \dim V$
 - $\dim V = \dim W$
 - None
- Dimension of vector subspace $W = \{(a, b, c) : a = b = c\}$ over vector space $R^3(R)$ equals
 - 0
 - 1
 - 2
 - 3
- If $U = \{(x, y, z) : x = y = z\}$, $V = \{(x, y, z) : x = 0\}$. Then what is $U + V$?
 - xy- plane
 - yz- plane
 - zx- plane
 - R^3
- If I is an identity transformation on finite vector space V . Then nullity of I is
 - $\dim V$
 - Zero
 - 1
 - None

8. Let V be a finite dimensional space. T is a zero transformation on V . Then range of T is
- V
 - $\{0\}$
 - ϕ
 - None
9. Let V be a vector space. T is a linear transformation of V into V such that $T(\alpha) = \alpha, \alpha \in V$ then T is
- Identity transformation
 - Zero transformation
 - Invertible transformation
 - Orthogonal transformation
10. Which of the following is a 2-dimensional subspace of R^3 over R
- $\{(0, x, 0) : x \in R\}$
 - $\{(0, x, 0) : x \in R\} \cup \{(0, 0, y) : y \in R\}$
 - $\{(x, y, 0) : x, y \in R \text{ and } x + y = 0\}$
 - $\{(0, x, z) : x, z \in R\}$
11. A set containing linearly dependent set is
- Linearly independent
 - Linearly dependent
 - Null set
 - None
12. If α is a characteristic root of a non-singular matrix A , then characteristic root of $\text{adj}(A)$ is
- $\alpha |A|$
 - α
 - $\frac{|A|}{\alpha}$
 - $\frac{|\text{adj}A|}{\alpha}$
13. Let P be a 4×4 matrix whose determinant is 10. The determinant of the matrix $-3P$ is
- 810
 - 30
 - 30
 - 810
14. Let P and Q be two $n \times n$ non-zero matrices such that $P+Q=0$. Which one of the following statements is never true?
- P is non-singular
 - $P = Q^T$
 - $P = Q^{-1}$
 - $\text{Rank}(P) \neq \text{Rank}(Q)$
15. The number of values of λ for which the system of equations
- $$\lambda x + (\lambda + 3)y = 10z$$
- $$(\lambda - 1)x + (\lambda - 2)y = 5z$$
- $$2x + (\lambda + 4)y = \lambda z$$
- has infinitely many solutions, is
- 1
 - 2
 - 3
 - infinite

16. An orthogonal set of non-zero vectors is
- a. Linearly independent
 - b. Linearly dependent
 - c. Constant
 - d. None
17. The orthogonal complement of inner product space V is
- a. Zero subspace
 - b. V itself
 - c. Any subspace
 - d. None
18. The Cauchy-Schwarz inequality states
- a. $|\langle \alpha \beta \rangle| \geq \|\alpha\| \|\beta\|$
 - b. $|\langle \alpha \beta \rangle| \leq \|\alpha\| \|\beta\|$
 - c. $\|(\alpha + \beta)\| \geq \|\alpha\| \|\beta\|$
 - d. None
19. Let S be an orthonormal set then for $\alpha \in S$
- a. $\|\alpha\| = 0$
 - b. $\|\alpha\| > 0$
 - c. $\|\alpha\| = 1$
 - d. $\|\alpha\| < 1$
20. If V be a vector space then V is
- a. Field
 - b. Abelian group
 - c. Null set
 - d. None

(Descriptive)

Time : 2 hrs. 30 min.

Marks : 50

[Answer question no.1 & any four (4) from the rest]

1. Find the eigen values and eigen vectors of the matrix 10

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

2. State Cayley-Hamilton theorem. Find the characteristic equation of 1+7+2
=10

the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and verify Cayley-Hamilton theorem.

Hence evaluate the matrix equation

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$$

3. Prove that a linearly independent subset of a finitely generated vector space is either a basis or can be extended to form a basis. 10

4. If T is an operator on R^3 whose basis is 10
 $B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ such that

$$[T : B] = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}. \text{ Find the matrix of } T \text{ with respect to basis}$$

$$B_1 = \{(0, 1, -1), (1, -1, 1), (-1, 1, 0)\}$$

5. Show that the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ is diagonalizable. 10

6. Verify whether the transformation is linear from R^2 into R^3 . Find the range, rank, null space and nullity for the transformation 2+2+4+
2=10

$$T(x_1, x_2) = (x_1 + x_2, x_1 - x_2, x_2)$$

7. State and prove Cauchy-Schwarz's inequality. 10

Or

If $C(x) = (x-3)^3(x-2)^4$ and $m(x) = (x-3)^2(x-2)^2$, then find all possible Jordan canonical forms of the given characteristic and minimal polynomials.

8. a. Show that the vectors $(0, 1, -2), (1, -1, 1), (1, 2, 1)$ form a linearly independent set. 5+5=10

- b. Find the dimension of the solution space W of the system of linear equations

$$x + 2y - 4z + 3r - s = 0$$

$$x + 2y - 2z + 2r + s = 0$$

$$2x + 4y - 2z + 3r + 4s = 0$$

= * * * =